SEARS FOUNDATION FOR MARINE RESEARCH
BINGHAM OCEANOGRAPHIC LABORATORY, YALE UNIVERSITY

JOURNAL OF
MARINE RESEARCH

VOLUME II 1939 NUMBER 1

NOTES ON THE DETERMINATION OF DISSOLVED OXYGEN IN SEA WATER

By

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The oxygen dissolved in sea water is usually determined by the classical method of Winkler. Probably because of the large number of investigators who have used this method, there have developed various modifications in the method of collecting samples, in the standardization of the sodium thiosulfate solution, and in the titration of the samples. It is the purpose of this paper to discuss these various methods, the sources of possible error, and to present the best procedure for standardization of the sodium thiosulfate solution.

COLLECTION OF SAMPLES

Large errors may result in the collection of samples from waters containing plankton in great abundance. This effect is limited to the first few meters of the surface waters. It is well known that the phytoplankton occur in colonies so that consecutive hauls of the net through the same stratum of water at approximately the same localities may result in variations in the quantitative measurements of plankton amounting to as much as 200 to 300 per cent. Since there is a direct correlation between the plankton content of water and its content of dissolved oxygen, wide fluctuations must result in the distribution of this element in these waters. The classical method of sampling may or may not give results that approximate average conditions. For detailed studies, particularly during very calm weather and in localities, such as fiords where there is marked stratification of the surface waters, proper sampling apparatus has yet to be devised.

* Contribution No. 86.

It is customary to collect the dissolved oxygen sample from the sampling bottle before those for other constituents are taken and as soon as the bottle comes aboard. This is essential in order to prevent possible expulsion of dissolved gases by increase in temperature. Furthermore, immediate sampling reduces the length of time the water is exposed to the metal of the sampling bottle. The larger the bottle the less metal exposed per unit volume of water. The authors utilize 2.5-liter sampling bottles of the Nansen type prepared by the Atlas Instrument Company of Seattle. To eliminate possible errors caused by the action of the dissolved gasses with the usual metal or alloy of the sampling bottles, silver plating was applied to the interior, but over a period of time this plate is acted upon by the sea water. Experiments are now in progress using stainless steel bottles.

The dissolved oxygen is determined with water samples contained in small glass-stoppered bottles which have been filled with water from the sampling bottles without loss or addition of oxygen. These small bottles should be made of amber or green glass so that light does not liberate iodine from the excess of the iodide reagent. Glass bottles having an approximate capacity of 100 or 250 ml. are commonly employed. The 100-ml. bottles contain only enough sample for a single titration if 0.01 N. sodium thiosulfate is used. Moreover they must be calibrated "to deliver" or "to contain" with an accuracy of 0.5 ml. to have an accuracy comparable with that of the titration. The calibration value for the bottle must be used in making the calculations for the oxygen content of the water; thus the volume of the bottle can not be incorporated in the constant for the equation when the results of a large number of titrations are being calculated as each bottle may have a different capacity. In order to facilitate calculations, to provide for a check titration, and to lessen errors caused by the addition of reagents and impurities contained in them, glass sample bottles having a capacity of 250 ml. are to be preferred. With such a procedure, a 100-ml. sample is taken for titration. The volume of the bottle is not involved in the size of the sample titrated and need not be known with any great accuracy. The calibration value of the bottle is only used in correcting for the dilution due to the addition of the reagents, 1 ml. for the manganese sulfate solution and 1 ml. for the potassium hydroxide-potassium iodide reagent.

$$100\ ml. \times \frac{250\text{–}2}{250} = \text{Volume Titrated}$$

Consequently a constant value of 250 ml. may be used as the volume of the bottle in the calculations even with bottles varying by 25 ml. in capacity and incur an error no greater than one part per thousand. If the calibration volumes of 100-ml. bottles varying by less than even 1 ml. are ignored in the calculations, the results would be materially affected. If the 250-ml. bottles are used, the best procedure is to calibrate a large number of the

bottles and then discard those with extreme maximum or minimum capacities.

When samples are kept in acid condition for some time, a small bubble of gas invariably collects at the stopper. Whether this is carbon dioxide liberated from the sample by the acid, nitrogen made more insoluble by the presence of the added electrolytes, or air taken in around the stopper due to thermal changes, is of little importance when aliquots of the solution are to be measured for the titration. When the entire content of the bottle is to be used for a single titration as is usually done with the 100-ml. bottle, the volume of the bottle does not represent the volume of the sample because of the presence of the gas bubble and an appreciable error ensues.

ESTIMATION OF DISSOLVED OXYGEN

Samples should be treated with the reagents as soon as possible after sampling. This is extremely important with waters which are saturated or supersaturated *in situ* and with the air temperature warmer than the water temperature. Even with unsaturated samples, the dissolved oxygen content can change on standing through biological processes.

Warming of the water sample during settling of the manganous-manganic hydroxide has little effect. Alternate warming and cooling is undesirable as air is sucked into the bottle, absorbed by the solution, and the oxygen extracted by the manganous hydroxide. Furthermore, the manganic hydroxide apparently ages on standing and dissolves quite slowly when later treated with acid. That is, it is undesirable to let the manganese hydroxides stand for a long period of time before acidification. If the samples are to be collected some time before the final titration is made, it is better to treat the samples with all the reagents and let the samples stand in the dark in the acid solution until titrated.

The 100-ml. portions need not be measured with a pipet but may be taken with a calibrated measuring cylinder. An error in measurement of the solution of only 0.5% results through such a practice. The cylinder is far more rapid and convenient to use than the pipet and is particularly appreciated while making titrations in heavy weather. Moreover, pipetting iodine solutions even with an accurate pipet is not without error as it causes a loss of iodine through volatilization. A constant technique of manipulating the graduated cylinder should be adhered to.

Hydrolytic Effect.—In solutions of iodine and potassium iodide, the following equilibria exist:

$$\begin{array}{c} I_2 + H_2O \rightleftarrows HI + HIO \\ + \\ I^-_3 \rightleftarrows I^- \end{array}$$

With dilution a larger proportion exists in the form of the acids because of the mass action effect. In like manner the addition of hydrogen ions decreases the proportion of hydriodic and hypoiodous acids. The addition of iodide ions decreases the concentration of free iodine as well as the hypoiodous and hydroiodic acids through formation of I_3^-. In the titration of dilute iodine solutions, it is important that this hydrolytic effect be kept at a minimum to insure a correct endpoint. This is effected, in this particular determination, by the presence of a sufficient excess of iodide ions which have been introduced previously by the hydroxide-iodide reagent.

REAGENTS

Manganous Sulfate.—The manganous sulfate should always be tested for the presence of ferric salts as impurities; if present, the salt is purified by crystallization.

Potassium Hydroxide-Potassium Iodide.—The most common interfering impurity in this reagent is iodate, which is often present in potassium iodide. Upon acidifying a portion of the reagent, after dilution with water, the iodate reacts as follows:

$$IO_3^- + 5I^- + 6H^+ \rightarrow 3I_2 + 3H_2O$$

Thus the liberation of iodine by the iodate will cause abnormally high titration results.

It is essential that this reagent should always be kept in a dark bottle to protect it from the action of light and thus prevent the formation of hypoiodites; a bottle coated on the exterior with black paint proves very efficient. Should the reagent be kept for longer than one month, it should be tested occasionally for hypoiodites.

$$IO^- + I^- + 2H^+ \rightarrow I_2 + H_2O$$

Sulfuric Acid.—Only the special analytical grade of sulfuric acid, free of nitrogen compounds should be used. Even C.P. acid may contain oxides of nitrogen if originally prepared by the Chamber Process, so that iodine is liberated when the diluted acid is treated with potassium iodide.

$$NO_2 + 2I^- + 2H^+ \rightarrow NO + I_2 + H_2O$$

On the other hand, the acid may contain traces of sulfur dioxide resulting from the reaction with organic matter such as dust of the air or accidental contact with rubber. Under such conditions, free iodine will be reduced and low results obtained.

Reagent Blank.—The magnitude of these impurities may be easily tested by adding, to 100 ml. water, the various reagents in the reverse order in which they are employed in the actual determination of dissolved oxygen.

That is, 1 ml. sulfuric acid is added to the water, followed by 1 ml. of the hydroxide-iodide reagent. At this point, the solution should be distinctly acid, then 1 ml. manganous sulfate is introduced. After standing for ten minutes, starch indicator is added, and any liberated iodine titrated with the thiosulfate. Blank determinations should give only very small values. If no iodine is liberated, then a drop of 0.0100 *N* KI-I_2 is added to the solution. The blue starch-iodide color should appear immediately. If a number of drops of the iodine solution are added before the blue color appears, undesirable sulfuric acid is thus shown. Either a fresh supply must be obtained or the reagent must be heated and allowed to fume copiously for a short period.

RECOMMENDED PROCEDURE FOR THE STANDARDIZATION OF SODIUM THIOSULFATE SOLUTION

In standardizing the thiosulfate solution, it is well to consider many of the factors to be discussed subsequently as well as those discussed previously. The following is the recommended procedure: To one hundred ml. of the average sea water under consideration, add 1 ml. of the sulfuric acid, followed by 1 ml. of the hydroxide-iodide reagent. After stirring introduce 1 ml. of the manganous sulfate solution. The resulting solution is then treated with an appropriate volume of standard solution, the liberated iodine titrated and the normality of the thiosulfate solution calculated. If the sea water contains excessive amounts of plankton or some reducing impurity, a distilled water medium is better used.

INVESTIGATION OF METHODS OF STANDARDIZATION

The importance of the standardization of the thiosulfate solution for use in the estimation of dissolved oxygen cannot be over-emphasized. For this reason the following methods of standardization were examined very critically. The standard solutions used in this investigation were prepared according to the following directions:

Sodium Thiosulfate Solution.—About 35 grams of sodium thiosulfate are dissolved in 10 liters of distilled water which has been previously boiled to expel carbon dioxide. It is stored in an amber glass bottle out of contact with the carbon dioxide of the air. A few drops of CS_2 are added as a preservative. A solution so treated is very stable over a long period of time.

Sodium Arsenite Solution, 0.01000 N. As_2O_3 is further purified by sublimation. Dissolve 0.4945 gram in 10 ml. of *N* $NaOH$, make slightly acid, buffer with 10 grams of $NaHCO_3$ and dilute to a liter.

Iodine Solution, 0.0100 N.—Purify the iodine by sublimation. Weigh about 1.270 grams I_2 into a weighing bottle containing 4 grams of KI and 5 ml. of water. Dilute to a liter and calculate the exact normality.

Secondary Iodine Solution.—Standardize the previous solution against the standard arsenite solution and calculate the normality of the iodine solution.

Potassium Dichromate Solution, 0.01000 N.—Purify the $K_2Cr_2O_7$ by recrystallization, powder and dry at 130° C. for 30 minutes. Weigh 0.4903 gram, dissolve, dilute to a liter.

Potassium Bi-iodate Solution, 0.01000 N.—Purify the $KH(IO_3)_2$ by recrystallization. Dry and weigh 0.3250 gram, dissolve, dilute to a liter.

EXPERIMENTAL

The normality of the sodium thiosulfate solution was determined as follows: Ten ml. of the standard solution were measured into 90 ml. of either distilled or sea water as was appropriate. In the case of potassium dichromate and potassium bi-iodate, iodine was liberated by adding 1 ml. of 36 N H_2SO_4 and 1 gram of KI. The sodium thiosulfate solution was added from a 10-ml. micro-buret until the yellow color of iodine was barely discernible; the starch indicator solution was added, and the titration continued until the blue color disappeared.

The normalities as established against the various standards are given in Table I. These results represent the averages of values obtained independently by three different chemists, each using his own set of standard solutions and the same thiosulfate solution.

TABLE I

Normality of a Sodium Thiosulfate Solution as Determined with Several Standards

Standard	*Diluted with Distilled Water*	*Diluted with Sea Water*
Primary I_2	0.01394	0.01382
Secondary I_2	.01373	.01372
$K_2Cr_2O_7$	.01369	.01369
$KH(IO_3)_2$	.01379	.01379
Average	0.01379	0.01375

DISCUSSION

Accuracy.—The weighings and measurements were made with an accuracy ample for this investigation. The iodine was weighed to the nearest milligram, thus involving an error no greater than one part in twelve hundred. The other solids were weighed to the nearest 0.1 milligram which was a comparable accuracy. Calibrated Normax equipment was used for all volumetric measurements so that an error no greater than one part in five hundred was involved. The starch endpoint is definite to 0.01 ml. but a total error of about 0.02 ml. may result due to overrunning the endpoint.

With an 8-ml. portion of thiosulfate, this means a possible error of one part in four hundred.

Volume of Titration Medium.—When the iodine from 10 ml. of the standard solutions was titrated with thiosulfate solution without first diluting, inconsistent results were obtained. This was probably due to loss of iodine by volatilization, though in the case of the oxidizing standards the large hydrogen-ion concentration may have caused decomposition of the thiosulfate solution, which, of course, would result in subsequent irregularities. However, when the standard was diluted with 90 ml. of water before liberating iodine, consistent results were obtained.

Distilled Water Medium versus Salt Water Medium.—The normality of the sodium thiosulfate solution was determined with various standards both in distilled water and in sea water. In this manner it was not only possible to compare the accuracy of the various standards but also to demonstrate that the dissolved salts in the sea water had an inappreciable effect on the estimation of dissolved oxygen. Theoretically such a standardization is desirable, however, since by standardizing in the same manner and with the same conditions as the titration is made on the unknown, a more accurate determination is supposedly obtained through compensation of errors. The error most likely to result when using sea water is the reaction of iodine with the organic matter. Table I shows that essentially the same results were obtained for each of the standards regardless of whether the medium was distilled water or salt water. This might not be the case if the water contained exceptionally large amounts of plankton.

Comparison of Various Standards.—It is also seen from Table I that no significant difference is noted for the normalities that result from the use of the various standards although the primary iodine solution tended to give high results due to slight losses of iodine. The most consistent results were obtained with potassium bi-iodate while the use of potassium dichromate yielded data showing the greatest variation.

These results show that it is possible to accurately standardize the sodium thiosulfate solution against any of these standards. However, in the selection of the best standard it is not solely a question of accuracy but also a question of cost of the pure standard, the convenience of preparation and use of the standard solution, its stability, etc.

Iodine is not recommended as a primary standard because of the difficulty in weighing and transferring to a volumetric flask without loss due to volatilization. Rather, it is recommended that the iodine be prepared as a secondary standard and then be checked against a sodium arsenite solution. Arsenic trioxide may be purified easily by sublimation and makes an excellent standard solution that is quite permanent. Its use makes the iodine a secondary standard.

Potassium dichromate may be purified very simply. It, too, is extremely stable. However, it has the disadvantage of reacting somewhat slowly with the potassium iodide. Occasionally the liberated iodine may be titrated by the thiosulfate before the dichromate-ion has reacted completely with the iodide. Therefore it is advisable to add additional acid just before reaching the endpoint to facilitate a complete reaction.

The potassium bi-iodate also makes an excellent standard. Moreover it reacts with iodide more rapidly than does dichromate. It is a rarer chemical than the others, is more difficult to purify on account of its greater solubility, and its cost is considerably greater.

SUMMARY

1. It is recommended that samples approximating 250 ml. of sea water be taken for dissolved oxygen and 100 ml. aliquot used for titration.

2. Potassium bi-iodate or a solution of iodine standardized against arsenious oxide is to be preferred for standardizing sodium thiosulfate.

3. The standard against which the thiosulfate is titrated should be placed in a volume of sea water approximating that used in the actual determination of dissolved oxygen, the sea water having been treated previously with the sulfuric acid reagent, then with the hydroxide-iodide reagent and manganous sulfate solutions.

TABLES FOR SIGMA-T

By

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It is frequently necessary to plot σ_t curves (equal density at zero pressure) as functions of the temperature and salinity. In such cases it is desirable to know the values of temperature and salinity corresponding to even intervals in density. The following tables have been found to be very useful when it is necessary to enter σ_t curves on temperature-salinity diagrams.

Table I has been prepared to show the salinity corresponding to units of σ_t and temperature and covers the range of σ_t between temperatures of $-2°$ and 30° and salinities between 22.40 °/₀₀ and 41.55 °/₀₀. The values are for units of σ_t between 18.00 and 30.00. The data in the table were obtained from Knudsen's "Hydrographical Tables" (1901), where on pages 43–46 are entered values of D in the following relationship:

$$\sigma_o = \sigma_t + D$$

for units of σ_t and temperature. From this relationship the values of σ_o corresponding to given values of σ_t and t were obtained. The required salinity value was then obtained from σ_o.

The tabulated values were checked by plotting and by determining the isothermal salinity differences corresponding to unit differences in σ_t. When the latter check was made, small irregularities were found which were removed by adjusting the salinity values. These changes were never greater than 0.02 °/₀₀.

In order to construct curves for fractional values of σ_t, linear interpolations between the salinity values on any isotherm may be made. Table II has been prepared for this purpose. It is necessary to make the interpolations for every few degrees as the salinity intervals increase with the temperature.

Assistance in the preparation of this material was furnished by the personnel of Works Progress Administration Official Project No. 465-03-3-403.

* Contribution from the Scripps Institution of Oceanography. New Series No. 59.

TABLE I

TABLES FOR σ_t

Values of the salinity (‰) corresponding to unit values of σ_t and temperature

Temp. σ_t	18.00	19.00	20.00	21.00	22.00	23.00	24.00	25.00	26.00	27.00	28.00	29.00	30.00
−2° C.	22.40	23.66	24.88	26.11	27.35	28.59	29.82	31.06	32.29	33.52	34.75	35.98	37.21
−1	22.40	23.64	24.88	26.12	27.36	28.60	29.84	31.08	32.31	33.55	34.78	36.02	37.26
0	22.41	23.66	24.90	26.14	27.39	28.63	29.87	31.11	32.36	33.60	34.85	36.09	37.33
1	22.43	23.69	24.94	26.18	27.43	28.68	29.93	31.18	32.43	33.68	34.92	36.16	37.41
2	22.48	23.73	24.99	26.24	27.50	28.76	30.01	31.26	32.51	33.76	35.01	36.26	37.51
3	22.55	23.80	25.06	26.31	27.57	28.83	30.09	31.35	32.61	33.87	35.12	36.37	37.62
4	22.62	23.88	25.15	26.41	27.67	28.93	30.19	31.46	32.72	33.99	35.25	36.50	37.75
5	22.72	23.98	25.25	26.52	27.78	29.05	30.32	31.59	32.86	34.13	35.39	36.65	37.91
6	22.83	24.10	25.38	26.65	27.92	29.19	30.47	31.74	33.01	34.27	35.53	36.80	38.06
7	22.97	24.24	25.52	26.79	28.07	29.35	30.62	31.90	33.17	34.44	35.71	36.98	38.25
8	23.12	24.40	25.67	26.95	28.23	29.52	30.79	32.07	33.35	34.63	35.90	37.18	38.45
9	23.28	24.57	25.86	27.14	28.42	29.70	30.98	32.26	33.54	34.82	36.10	37.38	38.66
10	23.46	24.75	26.04	27.32	28.61	29.90	31.18	32.47	33.75	35.04	36.32	37.60	38.88
11	23.66	24.95	26.24	27.53	28.82	30.11	31.40	32.69	33.97	35.26	36.55	37.83	39.11
12	23.86	25.16	26.46	27.75	29.04	30.34	31.63	32.92	34.22	35.50	36.79	38.08	39.36
13	24.09	25.39	26.69	27.99	29.29	30.59	31.88	33.18	34.47	35.76	37.05	38.34	39.62
14	24.33	25.63	26.94	28.24	29.54	30.84	32.14	33.44	34.74	36.04	37.33	38.62	39.91
15	24.59	25.89	27.20	28.50	29.80	31.10	32.41	33.71	35.01	36.31	37.61	38.90	40.19
16	24.85	26.16	27.47	28.78	30.08	31.39	32.70	34.00	35.30	36.60	37.91	39.21	40.50
17	25.14	26.45	27.75	29.06	30.38	31.69	32.99	34.30	35.61	36.92	38.22	39.52	40.81
18	25.43	26.75	28.06	29.38	30.69	32.00	33.31	34.62	35.93	37.23	38.54	39.85	41.15
19	25.75	27.07	28.38	29.69	31.01	32.32	33.64	34.95	36.26	37.57	38.87	40.18	41.48
20	26.07	27.39	28.71	30.03	31.35	32.66	33.97	35.29	36.60	37.91	39.22	40.53	
21	26.40	27.72	29.04	30.37	31.69	33.01	34.33	35.64	36.95	38.26	39.58	40.89	
22	26.75	28.07	29.40	30.72	32.05	33.37	34.69	36.00	37.32	38.64	39.95	41.26	

TABLE I—*Continued*

Temp.	σ_t *18.00*	*19.00*	*20.00*	*21.00*	*22.00*	*23.00*	*24.00*	*25.00*	*26.00*	*27.00*	*28.00*	*29.00*	*30.00*
23	27.12	28.44	29.76	31.09	32.41	33.74	35.06	36.38	37.70	39.02	40.34		
24	27.49	28.82	30.15	31.48	32.80	34.13	35.45	36.75	38.08	39.40	40.72		
25	27.87	29.20	30.53	31.86	33.19	34.52	35.85	37.17	38.49	39.81	41.13		
26	28.27	29.60	30.94	32.27	33.60	34.93	36.26	37.58	38.90	40.22	41.54		
27	28.68	30.02	31.35	32.68	34.01	35.35	36.68	38.01	39.33	40.65			
28	29.10	30.44	31.78	33.12	34.45	35.78	37.11	38.44	39.76	41.09			
29	29.54	30.88	32.22	33.55	34.89	36.22	37.55	38.88	40.20	41.53			
30	29.98	31.33	32.67	34.00	35.34	36.68	38.01	39.34	40.66				

TABLE II

INTERPOLATION TABLE FOR FRACTIONAL VALUES OF σ_t

Salinity difference	*0.1*	*0.2*	*0.3*	*0.4*	*0.5*	*0.6*	*0.7*	*0.8*	*0.9*
1.22 °/₀₀	0.12 °/₀₀	0.24 °/₀₀	0.37 °/₀₀	0.49 °/₀₀	0.61 °/₀₀	0.73 °/₀₀	0.85 °/₀₀	0.98 °/₀₀	1.10°/₀₀
1.23	.12	.25	.37	.49	.62	.74	.86	.98	1.11
1.24	.12	.25	.37	.50	.62	.74	.87	.99	1.12
1.25	.12	.25	.38	.50	.62	.75	.88	1.00	1.12
1.26	.13	.25	.38	.50	.63	.76	.88	1.01	1.13
1.27	.13	.25	.38	.51	.64	.76	.89	1.02	1.14
1.28	.13	.26	.38	.51	.64	.77	.90	1.02	1.15
1.29	.13	.26	.39	.52	.64	.77	.90	1.03	1.16
1.30	.13	.26	.39	.52	.65	.78	.91	1.04	1.17
1.31	.13	.26	.39	.52	.66	.79	.92	1.05	1.18
1.32	.13	.26	.40	.53	.66	.79	.92	1.06	1.19
1.33	.13	.27	.40	.53	.66	.80	.93	1.06	1.20
1.34	.13	.27	.40	.54	.67	.80	.94	1.07	1.21
1.35	.14	.27	.40	.54	.68	.81	.94	1.08	1.22

THE DETERMINATION AND OCCURRENCE OF ALUMINUM IN SEA WATER

By

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The presence of aluminum ions in sea water has long been an accepted fact, but there are few figures available on the concentrations in which the element exists and in particular under varying conditions. Thompson and Robinson (*7*) report certain results of analyses in which no attempt was made to prevent contamination of the samples with dissolved material from the bottles in which they were stored. The approximate concentration reported was 0.07 milligram atom per liter (1.90 mg.), a figure found to be in excess of the concentrations in samples which had been stored in bottles coated internally with paraffin.

It has been suggested that some of the aluminum in sea water is present in the form of organic complexes, and thus requires decomposition before ordinary analytical methods are adaptable. The variation, however, between values obtained directly and by wet combustion is quite small, indicating that most of the aluminum is probably present in the ionic form. By precipitation with 8-quinolinol (oxine) in neutral or slightly acid solution, interference by other elements in sea water is avoided.

The method employed was a modification of the colorimetric method of Alten, Weiland, and Loofman (*1*), using a Bausch and Lomb Universal spectrophotometer for the color comparison, thus limiting the error caused by visual peculiarities of the observer. A comparison of several suggested reagents was made before this method was adopted. Alizarin S, first proposed by Atack (*2*) was unsatisfactory. Cupferron (*3*) would be better suited to nephelometric analysis, although settling is quite rapid. Aluminon, most commonly used, required too careful control of conditions for use with sea water. Furthermore, iron is a disturbing factor, and the aluminum must be determined by an indirect analysis. Morin (*5*) might be used in fluorescence analysis but light transmission varies insufficiently. Kolthoff (*4*) suggests quinalizarin, but standard and sample must be run simultaneously, a process usually unnecessary in spectrophotometric work. There is also considerable tendency for the reagent itself to precipitate.

8-Quinolinol, 8-hydroxyquinoline, or oxine, precipitates aluminum as $Al(C_9H_7ON)_3$ from solutions of pH 4.2–9.8. Calcium and magnesium will

* Contribution No. 87.

precipitate at a *pH* above 8.2, and by regulation of the *pH* to 6–7, the interfering effect of these ions is negligible. Moderate amounts of aluminum can be determined by direct weighing of the complex, but for amounts such as are present in sea water, the colorimetric method (*1*) is better. The precipitated aluminum compound is dissolved in alcohol-*HCl* mixture and coupled with diazotised sulfanilic acid. The color which is developed with *NaOH* solution, follows Beer's Law, is stable for at least four hours, and is readily compared with a blank. As little as 0.002 mg. of aluminum gives a definite color reaction, although the errors involved in analysis of such small samples are much greater than more moderate amounts. It was necessary to use small samples of water as larger amounts give incomplete precipitation, and attempts to concentrate the sea water by radiant surface evaporation, with subsequent removal of the precipitating salts, gave low results. The results were about 0.01 mg. at./1. lower than samples analyzed directly.

REAGENTS

Oxine solution.—1 g. of the oxine is triturated with 1 ml. of glacial acetic acid and poured into 100 ml. boiling water. The solution is kept at boiling temperature with frequent stirring until the reagent has dissolved. The solution is then cooled and filtered.

Sodium acetate solution.—A saturated solution is prepared at about 50° and permitted to cool. The solution is kept in contact with excess solid sodium acetate.

Solvent.—Equal volumes of 2-normal *HCl* and 95% ethyl alcohol.

Sulfanilic acid.—8.6 g. are dissolved with warming in 1000 ml. of 30% acetic acid.

Sodium nitrite.—2.85 g. per 1000 ml. water.

Sodium hydroxide.—80 g. per 950 ml. water.

Impurities in the reagents should, of course, be avoided. Very small amounts can safely be compensated for in the calibration of the spectrophotometer with the standard solution and by comparison with the blank. The solutions are stored in paraffined bottles.

STANDARD

0.4397 g. of potassium aluminum sulfate is dissolved in water and made up to 1000 ml. 1 ml. of this solution contains 0.025 mg., equivalent to 0.00092 mg. at., of aluminum and is diluted to 5–10 ml. volume for use as standard. The extinction co-efficient of the standard is determined for use in calculating the values of subsequent analyses.

METHOD

The sea water sample is filtered through a Whatman No. 42 paper, and 10 ml: of the filtrate are pipetted into a Pyrex test tube. The sample is made faintly acid with 1 : 4; acetic acid : water (two drops usually enough), and 0.6 ml. of saturated sodium acetate solution added, followed by 0.25–0.50 ml. of the oxine reagent. The solutions are mixed by inversion and allowed to stand for 12 hours at room temperature. The tube is then placed in a water bath maintained at 70° for 30 minutes, after which the liquid is filtered through a medium-grained filter stick and the precipitate washed with successive small portions of boiling water. After washing, the filter stick is disconnected from the suction, and the sides of the precipitation tube are washed down with 2 ml. of alcohol-*HCl* mixture. The tube and filter stick are heated at 70° until the precipitate has completely dissolved, and the solution is then sucked through the filter stick which has not been removed from the test tube, into a 50 ml. volumetric flask. The test tube and stick are washed thoroughly with hot water which is also sucked through the stick into the flask. To the solution in the flask is added 1 ml. of sulfanilic acid solution and 1 ml. of sodium nitrite solution. The flask is allowed to stand for 10 minutes and 10 ml. of sodium hydroxide solution then added. The solution is made up to volume and after 10 minutes is transferred to the spectrophotometer tube and compared with a blank, prepared from the alcohol mixture, sulfanilic acid, sodium nitrite, and sodium hydroxide. If the aluminum concentration is too high, an aliquot is taken.

The solutions may also be compared on the colorimeter, preferably using a color filter. Nessler tubes are inconvenient as permanent standard solutions are not possible.

THE OCCURRENCE OF ALUMINUM

During the period of May 5, 1936, to May 5, 1937, surface samples of water were collected for the authors by Dr. Lyman Phifer in paraffined bottles from San Juan Channel from the observation pier of the field laboratories at Friday Harbor, Washington. Samples were taken at high tide every five days. Table I shows the average monthly concentrations of aluminum together with the maximum and minimum values found each month. The monthly averages show the general seasonal trend more accurately than the other data which fluctuates considerably.

There appears to be but a slight variation in the seasonal samples which are somewhat lower than the deep sea samples discussed later. The early spring months seem to give samples containing a larger amount of aluminum, possibly due to the leaching of suspended matter brought down by the rivers during the spring freshets. The wider variation between individual samples at that time of year also appears to support this statement.

The aluminum content of samples taken on August 27, 1936, in the Northeast Pacific Ocean off Cape Flattery from 50–100 miles off the continental shelf, were also determined. The results are shown in Table II.

The values obtained vary, increasing with depth with indications of maximum aluminum at 900 to 1000 meters. Below these depths the concentration decreases and as the bottom is approached increases again. This fact has also been observed in the case of iron as reported by Thompson and Bremner (*6*) and corresponds to the region of low oxygen values reported by Thompson, Thomas and Barnes (*8*).

Attempts were made to determine beryllium. The results in general were negative due to the lack of a sufficiently sensitive method.

SUMMARY

The concentration of aluminum in sea water has been determined.

The concentration of aluminum in sea water averages, for all samples, seasonal and depth, 0.020 mg. at./l., or 0.54 mg.

The monthly averages show a slight tendency to higher values in the early spring months and extending into June.

The concentration of aluminum varies with depth, there being a slight increase in the region approaching 1000 m., that region in which high iron and low oxygen values were also found.

TABLE I

Seasonal Variations in Aluminum Content of Sea Water in San Juan Channel

Milligram Atoms per Liter

1936	*Average*	*Maximum*	*Minimum*
May	0.016	0.021	0.012
June	.014	.024	.009
July	.013	.024	.006
August	.012	.018	.007
September	.009	.011	.007
October	.008	.010	.006
November	.009	.011	.007
December	.008	.012	.006
1937			
January	.009	.011	.006
February	.009	.011	.007
March	.009	.013	.008
April	.012	.016	.008
May	.015	....	....
Mean	.011	.015	.007

TABLE II

CONCENTRATION OF ALUMINUM AT VARIOUS DEPTHS IN THE WATERS OFF THE CONTINENTAL SHELF, NORTHEAST PACIFIC OCEAN

Milligram Atoms per Liter

Depth Meters	*Lat. 48° 06′ Long. 126° 30′*	*Lat. 48° 06′ Long. 127° 00′*
0	0.021	0.015
10	.019	.016
25	.016	.023
50	.018	.014
100	....	.012
150	.015	.013
200	.027	.018
300	.050	.024
400	.033	.028
500	.021	.033
600	.022	.047
700	.025	.054
800	.043	.033
900	.053	.061
1000	.055	.042
1100	....	.040
1200	.056	.020
1500	.048	.020
2000	.065	.027

REFERENCES

1. ALTEN, F., WEILAND, H., AND LOOFMAN, H.
1933. Angew. Chem. *46*, p. 668.

2. ATACK, F. W.
1915. J. Soc. Chem. Ind., *34*, p. 936.

3. BAUDISCH, O., AND ROM, H.
1916. Ber., *49*, p. 203.

4. KOLTHOFF, I. M.
1928. J. Am. Pharm. Assoc., *17*, p. 360.

5. SCHANTL,
1924. Mikrochemie, *2*, p. 174.

6. THOMPSON, T. G., AND BREMNER, R. W.
1935. J. du Conseil, *10*, p. 45.

7. THOMPSON, T. G., AND ROBINSON, R. J.
1932. "Physics of the Earth" V, Oceanography, Chap. V, National Research Council, Washington.

8. THOMPSON, T. G., THOMAS, B. D., AND BARNES, C. A.
1934. James Johnstone Memorial Volume p. 203–234, University Press of Liverpool.

VARIATIONS OF SEA LEVEL ON THE PACIFIC COAST OF THE UNITED STATES

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Seasonal Sea Level Variations.—The mean sea level during any period can be derived from the average of the hourly tide heights. These tide heights are obtained from records of the United States Coast and Geodetic Survey permanent tide gauges, which are located in the principal harbors along the west coast, except the La Jolla gauge which is situated on the open coast.

Marmer (1927) has pointed out that the mean monthly values of sea level show an annual cycle. At Seattle, low sea level occurs during the spring and summer, and high sea level during the winter. At San Francisco the sea level shows a semiannual period. At Los Angeles, La Jolla and San Diego, monthly means usually follow a single annual period with low values from March to May, and high values from July to September. In each case the range in monthly sea level during the year is small, seldom exceeding a foot, and when averaged for many years is less than half a foot.

Meteorological Tides.—Tidal experts recognize these seasonal fluctuations in sea level and describe them as long period meteorological tides. Harris (1907) and Shureman (1924) attribute their existence mainly to periodic changes in meteorological conditions, and to a very small extent to astronomical causes. The latter has been disregarded in the following consideration of the sea level fluctuations.

Meteorological tides formerly were believed to be due primarily to variations in atmospheric pressure and wind. However, Nomitsu and Okamoto (1927) attributed about 75% of the fluctuations in sea level on the Japanese coast to variations in the density of the water and 25% to variations of the atmospheric pressure. Montgomery (1938) has classified the most probable factors influencing sea level, and concluded that those which he terms *dynamic, wind, thermal* and *wide-spread effects* are the most important on the Atlantic coast. As the causes of sea-level fluctuations differ from place to place, some of the foregoing factors, particularly the density of the water, will be considered in connection with the sea level on the west coast.

* Contribution from the Scripps Institution of Oceanography. New Series No. 57. Acknowledgement is made of assistance by the personnel of Works Progress Administration, Office Project No. 465-03-3-403.

Density.—In order to show the effect of density upon sea level in southern California, vertical series of temperatures and salinities were examined from an oceanographic station located only five miles off La Jolla, which was occupied during the summer months of 1924 to 1927. The dynamic height differences which reflect the variations in the vertical distribution of density should indicate the fluctuations in sea level caused by changes in density.* The greatest common depth of repeated observations at this station is but 75 meters, yet the change in dynamic height in this upper layer accounts for a large part of the variation in sea level. This is evident from Figure 1,

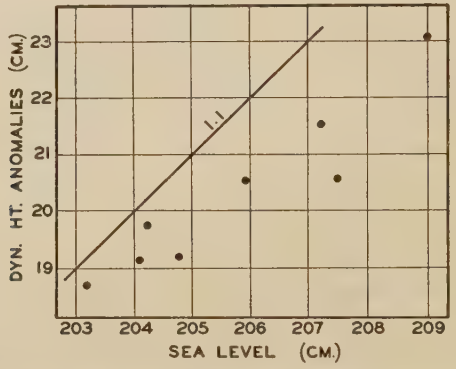

Figure 1. Relationship between the dynamic height anomalies (0 over 75 decibars) for an oceanographic station five miles west of the La Jolla tide gauge, and the mean sea level at La Jolla for two-week periods. Both are expressed in centimeters. The diagonal line represents a theoretical ratio in which the dynamic height anomalies correspond exactly to the sea level.

in which bi-weekly averages of the anomaly of the dynamic height above the 75-decibar surface are plotted against the corresponding average sea-level values according to the registrations at the La Jolla pier.

Recent unpublished data from the "*Bluefin*" and "*E. W. Scripps*" furnish oceanographic observations to a greater depth. In 1937 the "*Bluefin,*" on three different occasions, occupied about 30 oceanographic stations off the coast of southern California, covering an area of approximately 35,000 square miles. In 1938 the "*E. W. Scripps*" reoccupied the same stations at intervals of two months. Charts of the dynamic topography (0 over 500 decibars) were examined, and the contours of equal dynamic height were extrapolated to the location of the La Jolla gauge (Figure 8).

* It should be noted that fluctuations in dynamic heights do not correspond exactly to a linear difference in elevation. They should be divided by g/10, i. e., 0.98, in order to convert them into linear units. However, this difference is so small that for most purposes it is unnecessary to apply the correction.

In most instances the contours run nearly parallel to the coast, which make such extrapolation somewhat doubtful. Consequently, instead of selecting an individual value for the dynamic height, a certain range of values was chosen. These ranges are shown on Figure 2, together with the weekly sea level at La Jolla. The agreement is generally very close, especially when it is taken into account that local winds may cause piling up or removal of water near the coast.

The values of dynamic topography are relative to a common isobaric surface which is assumed to be level, and to be at a fixed elevation with respect to the bottom, or other established reference. From the close agreement of the dynamic heights and the sea level (Figure 2) the 500-decibar

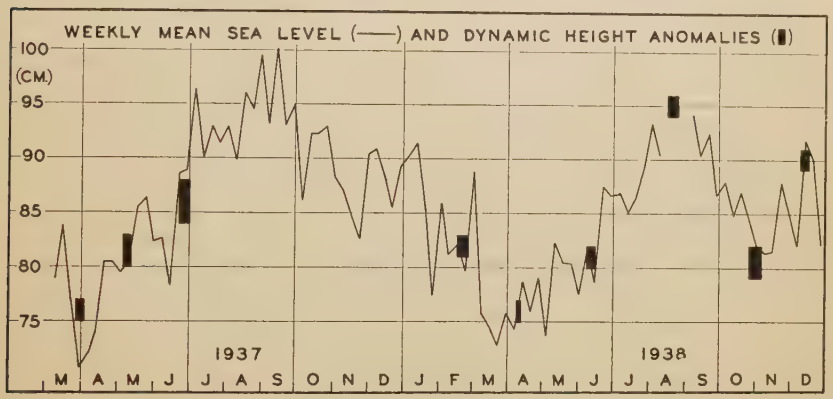

Figure 2. Seasonal variations in weekly sea level and dynamic height anomalies at La Jolla. The values of the latter were obtained by extrapolating the dynamic topography determined by three cruises of the California State Fish and Game Commission vessel "*Bluefin*" in 1937, and six cruises of the Scripps Institution vessel "*E. W. Scripps*" in 1938.

surface may be considered to be level, and seasonal changes in its elevation to be insignificant. Therefore, the relative topography can, in this case, be considered as absolute topography.

Other factors may directly or indirectly influence the sea level. Among these are variations in atmospheric pressure, winds, rainfall, and geological features. However, the preceding illustrations have shown that, in this region, practically all the fluctuations in sea level may be accounted for by changes in the density of the water. In special cases these other factors may play a leading role in sea level fluctuations and should be considered.

Atmospheric Pressure.—Some investigators treat the effect of atmospheric pressure upon sea level as a water barometer, lowering the sea level 13.2 inches for each inch increase in atmospheric pressure. Dietrich (1937) concludes from his investigation on the Gulf Coast that this factor is too

large. Others consider that regional differences in pressure are necessary in order to change the sea level. In other words, the difference in barometic pressure between any two stations when multiplied by the pressure factor, is equivalent to the difference in sea level, other factors being equal. The seasonal differences of average monthly atmospheric pressure at San Diego are small, averaging only 0.18 inches. Year-to-year differences of pressure are considerably less than this amount (Figure 4). The correction for atmospheric pressure upon sea level in this region is, therefore, in all events small and has not been applied. Winds associated with pressure gradients will generally obscure the direct effect of such gradients.

Winds.—Winds, through their mechanical action upon the water surface, may affect its slope. They may also influence sea level by their acceleration of or displacement of currents. It was found that winds from the usual direction, west and northwest, having a velocity of over ten miles per hour, lower the daily sea level. However, it is probable that wind is mainly an indirect factor which influences sea level through its widespread effect upon currents. Currents will be discussed later.

Water Volume.—The local removal or addition of water to the sea undoubtedly influences its height. Rainfall less than 0.5 inches per week shows no correlation with sea level, but a greater rainfall occurring within a week usually increases it. Storms which occur simultaneously with rainfall are accompanied by winds and changes in barometric pressure, making it difficult to evaluate the extent of fluctuations in sea level for which each of these factors is responsible. However, the resultant change occurring with rainfall shows a slight positive correlation.

Geological.—Changes in the form of the shore line, as suggested by Johnson (1929), may have some effect upon the sea level where land boundaries are changing by the formation or removal of land constrictions in the neighborhood of the tide gauge. These processes are usually very slow and are not significant in this discussion.

Temperature and Salinity.—Temperature and salinity, the determining factors of density, also display an annual variation. The annual range of average monthly salinity at La Jolla is from 33.48 to 33.72°/$_{oo}$ which at an average temperature of 17.36° C., will correspond to range in density from $\sigma_t = 24.28$ to $\sigma_t = 24.47$. The range in average monthly temperatures, from 13.93° C. to 20.77° C., corresponds at an average salinity of 33.60°/$_{oo}$ to a much greater range in density from, namely, $\sigma_t = 25.14$ to $\sigma_t = 23.51$. Seasonal temperature variations, then, have the greater effect upon surface density variations. These are only sea surface observations, yet they give some indication of conditions in the upper layer in which the maximum seasonal variation in density usually occurs.

As density is the principal factor in controlling sea level at La Jolla, deviations in the surface temperature of the water should be an indication

of deviations in sea level. By correlating the weekly deviations from the average of the weekly surface temperatures for 13 years with the corresponding deviations from the mean sea level for the same period, we obtain a definitely positive correlation. This means that below-normal surface temperatures occur simultaneously with below-normal sea levels and vice versa, irrespective of astronomical tides, winds, atmospheric pressure, etc.

The coefficient of correlation, r, is a means of expressing the agreement of deviation of these two variables, surface temperature and sea level. The

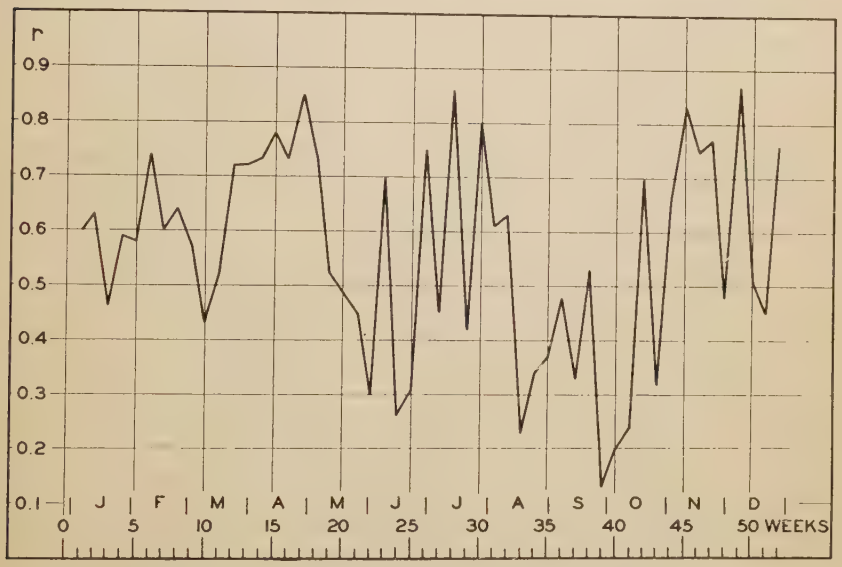

Figure 3. The coefficient of correlation r between the weekly departures of sea level and weekly departures of surface temperature at La Jolla for 13 years (1925–1937).

value of r is shown in Figure 3 for each week throughout the year. In 37 out of 52 weeks the probable error is less than one-third the coefficient of correlation. The best consistent agreement occurs in winter, yet every week the correlation of these short periods is positive. This further substantiates the evidence that density affects the sea level, and that temperature variations have the greatest effect.

Year-to-year Variation in Sea Level.—The material presented by Marmer (1927) and Shureman (1936) shows that, although there are some exceptions, in general the variations of sea level from one year to another at Seattle, San Francisco and San Diego are strikingly similar. These annual deviations from mean sea level which apparently affect the whole Pacific coast from Seattle to San Diego must be caused by widespread factors.

It can be seen from Figure 4 that during the past 17 years at La Jolla, for each case in which the average annual surface temperature is greater or less than the preceding year there is a corresponding rise or fall in the average annual sea level. The magnitudes of the changes are not always proportional to each other but the directions are the same. The coefficient of correlation by years is 0.67. Similar correlations have been made between the sea level and surface temperature at San Francisco, Los Angeles, and San Diego for an eleven-year period. The value of r for San Francisco is 0.77, Los Angeles 0.78, and San Diego 0.74.

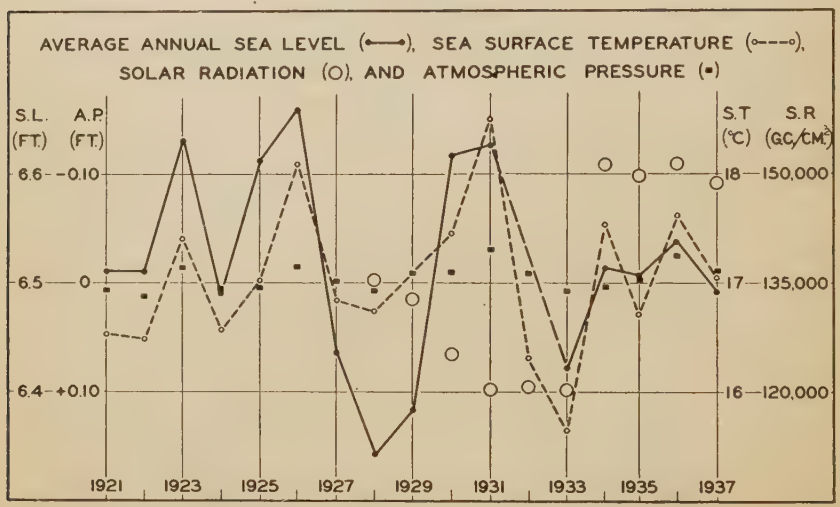

Figure 4. Relationship between (1), the average annual sea level (San Diego 1921–1924, La Jolla 1925–1937); (2), annual surface temperature (La Jolla); (3), total annual solar radiation (La Jolla); and, (4), the annual deviation from the average annual atmospheric pressure (San Diego). The latter was obtained from the Monthly Weather Reviews.

The total amount of solar radiation (McEwen, 1938) received at La Jolla during each year is also shown in Figure 4. It is evident that the fluctuations in surface temperatures are not principally dependent upon variations in radiation, but must be attributed to the effect of currents. This argument is further strengthened by considering the differences in temperature and salinity. Variations in temperature and salinity responsible for the differences in dynamic height shown in Figure 1 can not be explained alone by differences in the solar heating and evaporation. Part of the annual variation in salinity must be due to evaporation during spring and summer. Differences in observed salinity in August might therefore be due to differences in the amount of heat received during the spring and summer, but if

we calculate the heat necessary to increase, by evaporation, the salinity of the stations occupied in August, 1925, to the salinity of the stations occupied in August, 1926, it is found that in the latter year 31,000 gram calories per square centimeter must have been received above the amount of 1925. As the maximum difference in solar radiation between any two consecutive years is practically always less than this amount, and as all solar radiation is not available for heating and evaporation, the water masses must have had different histories.

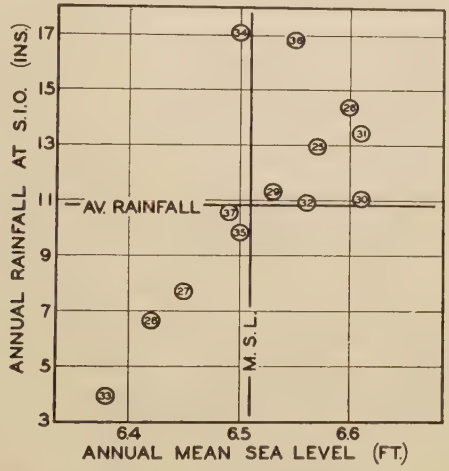

Figure 5. Relationship between the total annual rainfall at the Scripps Institution of Oceanography in inches and the annual mean sea level for southern California expressed in feet. The sea levels refer to the calendar year, the recorded rainfall to a year which begins six months later.

Atmospheric pressures for southern California are fairly constant, not only throughout the year, but also from year to year. In Figure 4, the deviations of mean annual pressure have also been represented. By multiplication with the factor 13.2, the observed values have been converted to correspond to sea-level changes. These deviations amount to about 10 per cent of the larger original sea level deviations. As previously stated, no corrections in sea level have been made for atmospheric pressure. However, it should be noted that in most cases the sign of the departure of pressure from the mean is opposite in sign to the sea level departure. Short period fluctuations in pressure may have greater effect upon sea level than those of longer periods, due to the greater ranges of pressure.

Further indication of widespread factors associated with sea-level changes is obtained by plotting the rainfall at the Scripps Institution in the winter season against the sea level of the preceding calendar year (Figure 5). The

calendar years for which the average sea levels were computed are shown in the circles.

The sea level used in the correlation is an average from the three southern California permanent tidal stations, Los Angeles, La Jolla, and San Diego. The period is for each calendar year since 1925, when the La Jolla tide gauge was installed.

The rainfall is also for a year, but corresponds to a period which begins six months later than the sea-level period. The latter is from January through December, while the rainfall period begins with July and extends

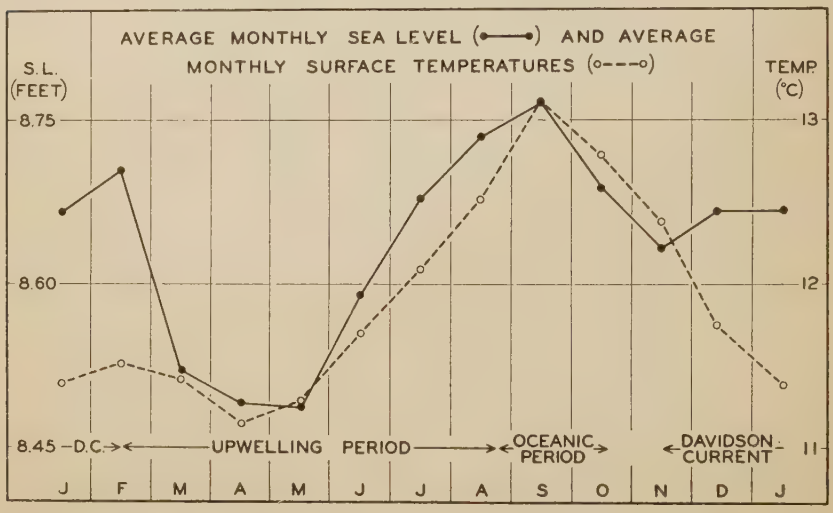

Figure 6. Relationship between the average monthly sea level at San Francisco, average monthly surface temperatures at the Farallon Islands (1925–1936), and currents off Monterey, according to Skogsberg.

through June. In southern California nearly all rain falls during the winter and it was, therefore, stated that the rainfall of the winter season was plotted against the sea level of the preceding calendar year. In practically every year since the installation of the La Jolla tide gauge, when the sea level was higher than normal, the following rainfall was above average. Likewise, subnormal sea level occurs previous to a below-average rainfall. This six-months lag suggests a means of rainfall prediction.

Currents.—The sea level at San Francisco, as previously stated, shows a semiannual variation, but winter surface temperatures at the Farallon Islands 25 miles west of San Francisco do not always correspond with the San Francisco sea levels (Figure 6). This difference can be better understood by considering the currents off Monterey Bay. Skogsberg (1936)

shows that during this period of disagreement, November to February, the "Davidson Current" extends to this region, introducing warm subsurface water without markedly increasing the surface temperature. The inflow of this warm subsurface water may account for the high sea level during the period of the Davidson Current. Immediately following this period a period of upwelling begins and the sea level is lowered, depending on the amount of upwelling. During the latter half of the upwelling period the density of the surface water decreases because of summer heating and the sea level rises. As the upwelling gradually ceases, and oceanic period character-

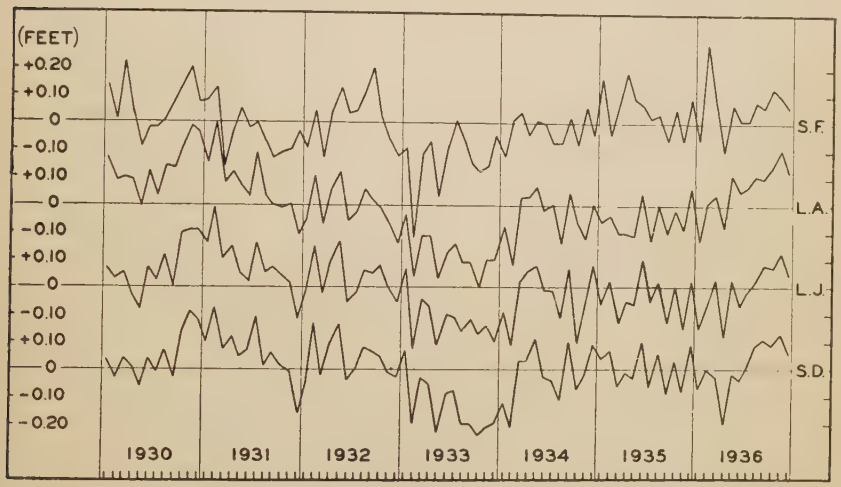

Figure 7. Departures of the monthly sea level values from seven years (1930–1936) monthly average for San Francisco, Los Angeles, La Jolla and San Diego.

ized by the inflow of warm offshore water takes its place. At this time the sea level reaches its maximum, due to the increased temperature, especially near the surface. Thus, the currents appear to be instrumental in producing temperature changes and, accordingly, sea-level changes. Causes of these changes in the currents are not yet fully understood.

It has been shown that it is necessary to take into account the difference in the character of density of the water associated with currents in order to account for the total changes in the temperature and salinity of water throughout the year. It follows that when the variations in the annual sea level at Seattle agree with those at San Francisco and San Diego similar alterations have occurred in the water columns over a wide area.

Departures from the average monthly sea level at San Diego, La Jolla, and Los Angeles are in remarkably close agreement (Figure 7), but the San

Francisco deviations do not always agree with those at the southern California stations. This is particularly true during the winter of 1934-35. When reasoning as before, these monthly departures would be representative of alterations due to currents which are effective in a smaller area and for shorter periods. Still smaller areas are probably being affected simultaneously by changes of even shorter duration.

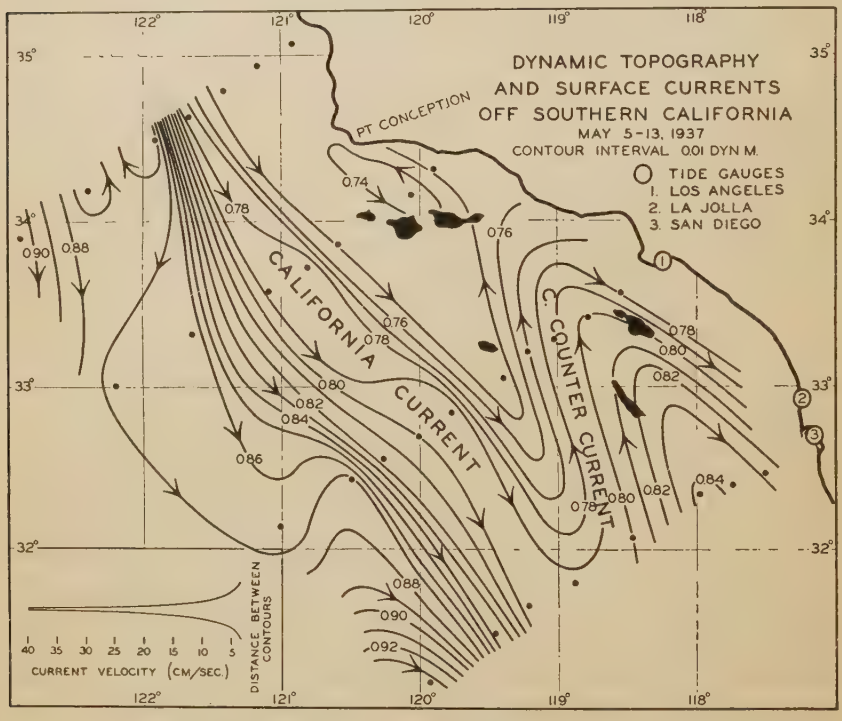

Figure 8. Dynamic topography and surface currents off southern California according to observations on the second cruise of the "*Bluefin*."

The currents recorded at the Scripps Institution Pier are mainly tidal, but the influence of eddies and offshore currents is also significant. In general, a resultant inshore current to the south lowers the sea level, while northerly inshore currents cause the sea level to rise. The change depends largely upon the differences in density of the water displaced, which is associated with changes in the direction and velocity of the current. Because currents influence sea-level variations in this region, factors which are attrib-

uted to currents, such as their erosive power, will show a high correlation with sea level (LaFond 1938).

The general circulation off the southern California coast has been investigated by the "*Bluefin*" and "*E. W. Scripps*," and some of the outstanding features have been described by Sverdrup (1938). In spring and early summer its main features usually consist of (1), the California Current flowing to the southeast or about parallel to the coast, (2), the California Counter Current nearer shore flowing to the north and sometimes turning and forming (3), an inshore current to the southeast (Figure 8). The last two currents may be thought of as the inshore part of a large eddy.

During periods when the Counter Current and the inshore current are about equal, as in Figure 8, the surface topography slopes down toward the coast. When the Counter Current becomes stronger and the inshore current is less pronounced or non-existent, the slope of the sea surface is reduced or may slope up towards the coast. In the former case, sea level at the coast is low and surface temperature near the coast is low, in the latter case both sea level and surface temperature near the coast are high. Thus, sea level is related to the character of the Counter Current. When this turns around such that the total inshore transport to the north is small, sea level is low, but when the total transport to the north is great, sea level is high. It is probable that in years of high sea level the northerly inshore flow is more pronounced and extends farther north than in years of low sea level.

The cause of this eddy is not clearly understood but it seems probable that the prevailing wind systems which produce the general oceanic circulation vary from year to year, thus causing changes of the currents which, in turn, are reflected in changes of the sea level.

SUMMARY

An examination of variations in sea level along the west coast of the United States and simultaneous variations of the density of the waters near the coast leads to the following conclusions:

1. Nearly all the variations in sea level on the West Coast of the United States can be accounted for by variations in the density of the water above 500 decibars.

2. The changes in density of the coastal waters are mainly explained by changes in the currents.

3. Current variations of long duration affect the sea level in large areas; variations of short duration affect a smaller area.

4. The variations of the currents appear to be caused by widespread variations in the atmospheric conditions. This conclusion is substantiated by the fact that variations in winter rainfall at La Jolla shows a high correlation with the variations in the annual sea level on the coast of Southern California.

Acknowledgements.—The author wishes to express his acknowledgement to Dr. H. U. Sverdrup and Dr. R. H. Fleming for suggestions and helpful criticism during the course of this work, and for placing at his disposal unpublished data from the cruises of the "*Bluefin*," and the "*E. W. Scripps*"; to Dr. G. F. McEwen for permitting the use of data, not yet published, on file in the division of dynamical oceanography at the Scripps Institution; and to the United States Coast and Geodetic Survey for supplying sea-level data which made this analysis possible.

LITERATURE

Dietrich, Gunter.

1937. Die Lage des Meeresoberflache in Druckfeld von Ozean und Atmosphare mit besonderer Berucksichtigung des westlichen Nordatlantischen Ozeans und des Golfs von Mexico. Veroffentlichungen des Instituts für Meereskunde an der Universität Berlin, Neue Folge, A, Heft 33, pp. 1–52, 1937.

Harris, R. A.

1897. Manual of tides, Parts 1–5, issued as Appendices to Annual Reports, U. S. Coast and Geodetic Survey for 1894–1907. Part II, p. 532, 1894.

Johnson, Douglas.

1929. Studies of mean sea-level. Report of the Committee on Shore-line Investigations, Nat. Research Counc., Bull. No. 70, p. 1–50, July, 1929.

LaFond, Eugene C.

1938. Relationship between mean sea level and sand movements. Science, vol. 88, no. 2274, p. 112–13, July 29, 1938.

Marmer, H. A.

1927. Tidal Datum Planes, U. S. Coast and Geodetic Survey, Spec. Pub. no. 135, p. 1–139, 1927.

McEwen, George F.

1938. Some energy relations between the sea surface and the atmosphere. Journ. Marine Research, vol. 1, no. 3, p. 217–38, 1938.

Montgomery, R. B.

1938. Fluctuations in monthly sea level on eastern U. S. Coast as related to dynamics of western north Atlantic Ocean. Jour. of Marine Research, vol. 1, no. 2, p. 165–185, 1938.

Nomitsu, Takaharu and Motojiro Okamoto.

1927. Causes of the annual variation of the mean sea level along the Japanese coast. Memoirs of the College of Science, Kyoto Imperial University, Series A., vol. 10, no. 3, p. 1–161, 1927.

Schureman, Paul.

1924. A manual of the harmonic analysis and prediction of tides. U. S. Coast and Geodetic Survey, Spec. Pub. no. 98, p. 1–413, 1924.

Schureman, Paul.

1936. Variation in yearly sea level, p. 23–25 in: Bowie, Geodetic operations in the United States, January 1, 1933, to December 31, 1935. U. S. Coast and Geodetic Survey, Spec. Pub. no. 207, p. 1–25, 1936.

Skogsberg, Tage.
1936. Hydrography of Monterey Bay, California. Thermal Conditions, 1929–1933. American Philosophical Society, Trans. vol. 29, p. 1–152, December 1936.

Sverdrup, H. U.
1938. Am. Geoph Un. 19th Ann. Meet., Trans., p. 238–42, 1938.
Research within physical oceanography and submarine geology at the Scripps Institution of Oceanography during April 1937 to April 1938.

PERIODIC CHANGES IN SPECTRAL SCATTERING AND SPECTRAL TRANSMISSION OF DAYLIGHT IN TIDAL WATER

By

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Seasonal changes in the transmission of certain spectral bands of solar radiation in the inshore waters of the Pacific Northwest were reported by Williams and Utterback (*1*) for the year 1934 and by Utterback and Miller (*2*) for the years 1935 and 1936. During each of these periods approximately one month elapsed between sets of observations, hence the results were computed from data taken at fairly long intervals of time. Also, in these same waters, some measurements on the scattering of certain spectral bands were made by Utterback and Jorgensen. (*3*) In view of the results of these observations it seemed desirable to make a series of weekly measurements of the simultaneous values of absorption and scattering in the same locality. The results of these measurements are here presented.

A location was selected west of Crane Island which is one of the islands in the San Juan group. The depth of the water at this station is approximately 35 meters. Throughout the series of observations very favorable conditions prevailed. The sky was unusually clear and the water was relatively smooth, even though tidal flow existed.

A submarine photometer, designed to simultaneously measure the relative intensities of the upward and the downward traveling light, was constructed of an eight-inch pipe closed at each end with 3/8 inch boiler plate caps. The upper cap was constructed so that it held two water-tight windows behind which were mounted two Photronic cells. These cells were connected and operated independently in order that any shadow effect of the cables and the cable supports might be detected. A large Electrocell, facing downward, was mounted behind a window in the lower cap. Identical filters, for defining the band of light admitted to the cells, were placed between each cell and its window.

The conducting cables were brought out through a water-tight gland and terminated on board the small boat from which the measurements were made. A deck photometer, also provided with identical filters, was used to observe the intensity of the light incident upon the surface.

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TABLE I

SPECTRAL EXTINCTION COEFFICIENTS PER METER

Depth Range, Meters	June 27	July 1		July 8		July 15	July 22		July 29	Aug. 5	Aug. 12		Aug. 19	
4600 Angstroms														
0–5				.241		.370	.316	.325	.522	.372	.517	.495	.363	
5–10				.232		.350	.310	.317	.494	.351	.491	.464	.325	
10–15				.223		.331	.307	.303	.508	.324	.480	.442	.315	
15–20				.222		.314	.297	.283	—	.314	—	.428	.317	
20–25				.218		—	—	.251	—	—	—	—	—	
4800 Angstroms														
0–5		.360	.386	.266		.322	.278		.446	.345	.474	.448	.324	
5–10		.368	.338	.238		.322	.266		.442	.315	.432	.438	.314	
10–15		.364	.346	.212		.344	.250		.446	.300	.412	.434	.318	
15–20		.358	.358	.230		.370	.262		.444	.300	.422	.432	.308	
20–25		—	—	.236		.358	.260		—	.282	—	—	.300	
25–30		—	—	—		—	.248		—	—	—	—	—	
5150 Angstroms														
0–5				.187		.279	.224		.225	.407	.274		.388	.246
5–10				.192		.280	.241		.239	.402	.274		.376	.252
10–15				.191		.280	.237		.246	.394	.271		.377	.249
15–20				.197		.283	.238		.257	.383	.275		.379	.250
20–25				.201		.283	.237		.257	.376	.277		.376	.245
25–30				.203		.269	.289		—	—	—		—	.239
5300 Angstroms														
0–5	.224	.272	.265	.198	.198	.241	.221		.326	.284	.349		.221	.253
5–10	.226	.278	.263	.193	.193	.249	.216		.335	.281	.330		.234	.264
10–15	.228	.280	.267	.167	.179	.316	.214		.331	.289	.335		.252	.273
15–20	.223*	.284	.266	.167	.165	.308	.212		.330	.267	.312		.248	.239
20–25	—	.290	.256	.181	.176	.310	.211		.328	.263	.299		.238	.233
25–30	—	—	—	.192	.188	.305	.221		.328	—	.298		—	—
30–33	—	—	—	—	.195*	.300*	.230*		—	—	—		—	—

TABLE I—Continued

Depth Range, Meters	June 27	July 1		July 8	July 15	July 22	July 29	Aug. 5	Aug. 12		Aug. 19	
6000 Angstroms												
0–5		.428	.428	.374	.448	.407	.480	.466	.503		.400	.464
5–10		.428	.428	.375	.455	.414	.473	.461	.497		.404	.453
10–15		.420	.422	.380	.448	.411	.465	.453	.475		.397	.439
15–20		.414	.414	.377	.445	.409	.466	.449	.470		.389	.424
20–25		.400	.412	—	.436	—	.464	—	—		.376	—
6600 Angstroms												
0–5						.675	.534	.826	.770	.706	.736	
5–10						.570	.698	.612	.472	.580	.600	
10–15						.428	.496	—	—	.512	.562	

* 3-Meter Interval.

TABLE II

Upward Light Intensity in Per Cent of the Downward Light Intensity

Depth in Meters	June 27	July 1		July 8	July 15	July 22	July 29	Aug. 5	Aug. 12		Aug. 19
4600 Angstroms											
0						.42	.92	1.68	1.76	2.64	3.13
5						.76	.95	1.92	1.85	2.46	3.39
10						.99	.98	2.10	2.44	2.38	2.52
15						1.14	1.00	2.25	4.06	2.61	2.70
4800 Angstroms											
0		.95	1.14	1.70	1.68	1.27	1.00	1.64	1.36	2.30	2.24
5		.69	.93	2.69	1.94	1.49	1.07	2.00	1.66	2.10	2.60
10		.70	.82	2.84	1.76	1.52	.86	1.80	1.84	1.96	2.24
15		.92	.84	2.84	1.75	1.48	1.02	1.75	1.82	1.89	2.13
20		2.20	1.17	3.14	2.12	1.57		1.88	1.88		2.17
25			4.6	3.02	2.37	1.97		2.59			

TABLE II—Continued

Depth in Meters	June 27	July 1		July 8	July 15	July 22	July 29	Aug. 5	Aug. 12	Aug. 19	
5150 Angstroms											
0						1.75	1.42	3.24	2.72	3.18	
5						1.98	1.44	2.43	2.80	3.05	
10						1.91	1.44	2.30	2.52	2.73	
15						1.96	1.58	2.34	2.51	2.20	
20						2.01	1.64	2.59	2.66	1.82	
25						1.90		3.29		1.74	
5300 Angstroms											
0	1.24	.91	1.16	1.99	1.67	1.13	.80	2.02	1.48	1.87	2.10
5	1.41	1.00	1.18	2.32	2.31	1.46	1.17	2.13	1.85	1.98	2.76
10	1.88	.65	.66	2.34	2.33	1.73	1.33	1.85	1.68	1.94	2.79
15	2.02	.77	.74	2.25	2.18	1.80	1.25	1.69	1.73	1.72	2.90
20		1.13	.96	2.07	2.18	1.63	1.09	1.93	2.09	1.66	
25			2.01	2.30	2.94	1.62	1.64	4.36	2.99	1.85	
30				3.64	4.57	2.65	3.58		4.70	3.92	
6000 Angstroms											
0		.30	.47	1.94	.49	.38	.47	.93	.78	.85	
5		.26	.56	2.01	.97	.45	.67	.96	.97	.86	
10		.31	.63	2.52	1.09	.62	.75	1.18	1.12	.84	
15		.62	.91	5.41	1.58	1.00	.92	1.28	1.39	.97	
20		1.65	3.09		2.55	2.26	1.40	1.40	2.38	1.34	
6600 Angstroms											
0						.08	.19	.43	.44	.26	.44
5						.64	.82	1.47	1.19	1.09	1.42
10						2.78	2.24	2.38	1.54	5.30	5.80

The method of making the measurements, the electrical instruments, the light filters, and the precautions taken have been previously described by the authors. (*3, 4*) However, in the observations here reported, the photometer was lowered to six inches beneath the surface of the water and then by subsequent intervals of five meters until the limit of the indicating apparatus was reached. The measurements were repeated, at the same depths, on returning the apparatus to the surface. The extinction coefficients, for each color were computed as previously described and the results given in Table I.

When two independent sets of data were taken on the same day for any particular spectral band they are recorded as independent sets in Table I. The extinction coefficients for six color bands were computed. However, as but three of these bands were employed during all of the measurements, only the variations in the coefficients corresponding to these bands are shown in Figure 9.

Just beneath the curves for each color filter is a curve representing the mean height of tide at the time observations were made. These curves have significance only at the points in question, the lines being drawn for visual aid. Small arrows are drawn near the plotted points to indicate flooding, ebbing or slack tides.

The intensity of the upward traveling light expressed in per cent of the downward traveling light is recorded in Table II.

Figure 10 represents the fluctuations of the scattered light of the three bands which were used during the entire series of observations.

Net hauls were made at the time and depth of each observation by Professor R. C. Miller who is making an analysis of the microscopic organism content in conjunction with the periodic changes shown in Figures 9 and 10.

The variations in the extinction coefficients, as shown in Figure 9, appear to be cyclic with high extinction on flooding tides and low extinction on ebbing tides. In view of this it is supposed that on flood tides the water runs from the east to west through Harney Channel carrying more or less stagnant water from West Sound and from East Sound past Crane Island to give high extinction of light. On the ebb tides the clear water from the Gulf of Georgia, to the north, flows south past Crane Island to give low light extinction. The observed flow of the tide is in agreement with this hypothesis.

During the observations any list of the boat and any errors in reading the position of the photometer resulted in an uncertainty in the depth which, for a single observation, might be as much as five per cent. Any changes in the transparency of the water during the measurements affect the values of the extinction coefficients. For each set of observations the data were taken with the photometer descending and again when it was ascending, and generally required but a few minutes. Since variations in the observed

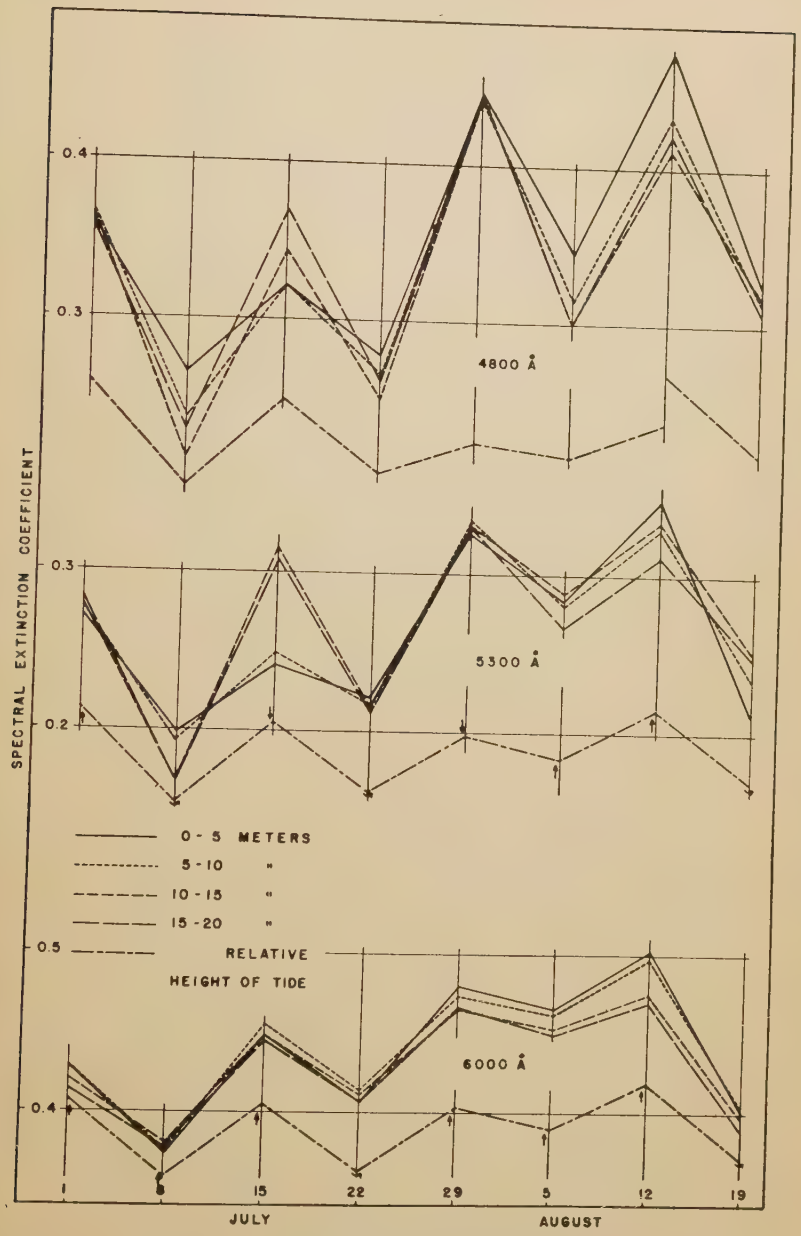

Figure 9. Variation of the extinction coefficients.

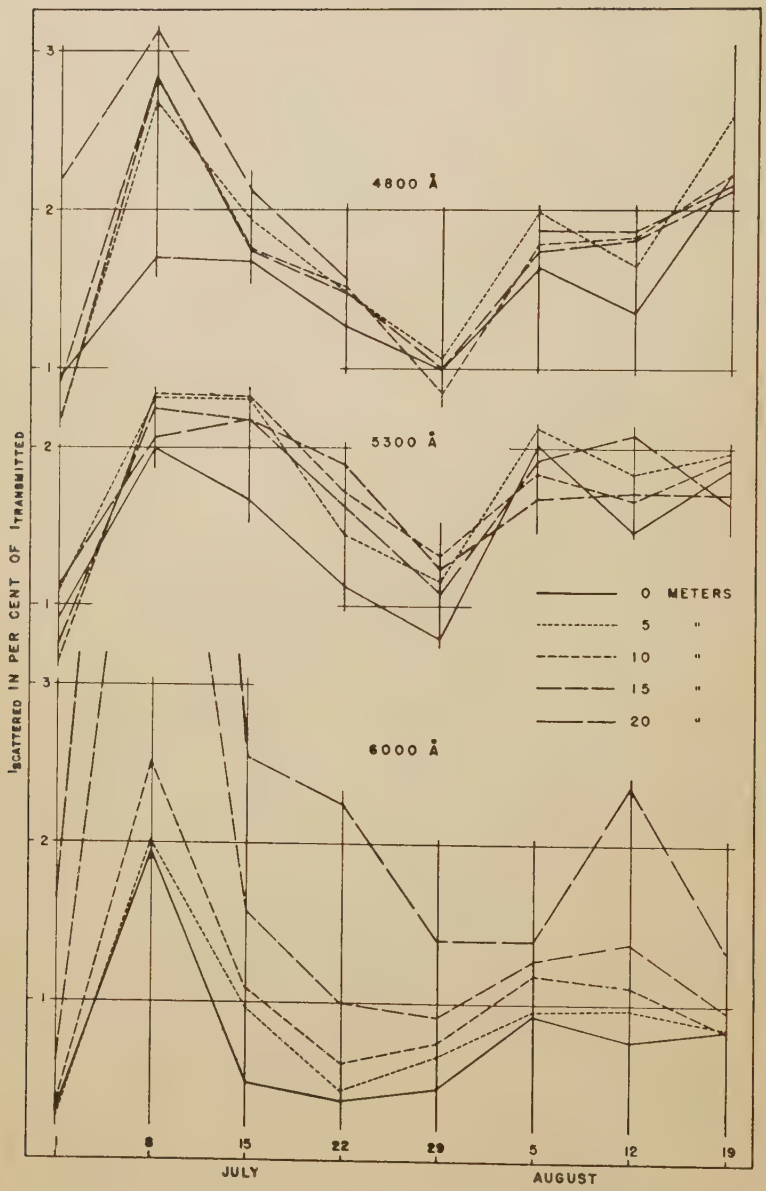

Figure 10. Variations in scattered light.

intensity were those primarily due to the change in water and variations in depth, the values obtained experimentally were found to be distributed about a mean geometrically instead of algebraically. The mean of duplicate sets of data was thus obtained by taking logarithms of the relative intensities and then determining the mean value algebraically.

The light scattered, in per cent of the light transmitted, was calculated for each individual observation. These ratios were found to be equally distributed on each side of the mean and so were averaged algebraically. Usually duplicate sets of individual data checked within three per cent.

The numerical values of the extinction coefficients ranged from 0.23 to 0.52 per meter for the 4600 Å band and from 0.48 to 0.83 per meter for the 6600 Å band, while for the 5300 Å band the variation was from 0.17 to 0.34 per meter. From the data in Table I it is found that variations in the extinction coefficients amounted to about 75 per cent of the mean for the 4600 Å band and only about 30 per cent of the mean for the 6000 Å band, which shows that the extinction characteristics of the short wave lengths are more dependent upon the turbidity of the water than are those of the long wave lengths. This variation is consistent with the observed shift in the region of maximum transparency from blue to green on coming to inshore waters from the open ocean. (*4*)

The relative intensities of the scattered light were found to range from 1 per cent to 3 per cent for the short wave lengths and from 0.5 per cent to 2 per cent for the long wave lengths. The relative intensities of the scattered light were found to increase in the immediate vicinity of the bottom in those instances where it was possible to make measurements very near the bottom of the water body.

REFERENCES

1. Williams, E. Allan and C. L. Utterback.
1935. Seasonal Changes in Components of Submarine Daylight. J. O. S. A. 25, 384,

2. Utterback, C. L. and Eugene K. Miller.
1937. Variations in Components of Submarine Daylight for 1935 and 1936. Jour. Du Conseil XII, 305.

3. Utterback, C. L. and Wilhelm Jorgensen.
1936. Scattering of Daylight in the Sea. J. O. S. A. 26, 257

4. Utterback, C. L. and Wilhelm Jorgensen.
1934. Absorption of Daylight in the North Pacific Ocean. Jour. Du Conseil IX, 197.

RELATION BETWEEN VARIATIONS IN THE INTENSITY OF THE ZONAL CIRCULATION OF THE ATMOSPHERE AND THE DISPLACEMENTS OF THE SEMI-PERMANENT CENTERS OF ACTION*

By

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This paper attempts to interpret, from a single point of view, several at first sight independent phenomena brought into focus through the synoptic investigations carried on at the Massachusetts Institute of Technology during the last few years. Since this interpretation is very largely based on a consideration of the changes in vorticity which must occur in vertical air columns which are displaced from one latitude to another and since such vorticity changes play a fundamental role also in Ekman's general ocean current theory (1932), the results would appear to be of enough interest to physical oceanographers to warrant their publication in this journal. The particular phenomena brought out in the course of our studies are listed below.

1. Mean monthly isentropic charts for the month of August from the last few years show that the atmosphere over the United States at that time of the year is characterized by a well-marked anticyclonic cellular structure, each cell or eddy consisting of a dry current from the north and a moist current from the south, the total diameter of each cell being from 1500 to 2500 miles (Namias and Wexler, 1938). Eddies of this type also represent the most outstanding feature of the *daily* isentropic charts for the summer season; the fact that the mean monthly charts show the same structure proves that the individual eddies are very nearly stationary and that they frequently regenerate in certain preferred locations. A comparison of the mean charts from different years shows that in spite of marked differences from year to year, one tongue of moist air always enters the United States

* This paper contains a preliminary report on certain phases of an investigation carried on at the Massachusetts Institute of Technology in cooperation with the U. S. Department of Agriculture, the objective being the development of adequate methods for weekly weather forecasts. The synoptic phases of this project, involving the preparation of daily weather charts for the Northern Hemisphere, is under the direction of Dr. H. C. Willett with the cooperation of Messrs. J. Holmboe and J. Namias. The statistical phases are under the direction of Mr. L. Page with the co-operation of Mr. R. Allen, both of the Bureau of Agricultural Economics.

from the southeast around or to the west of El Paso in New Mexico. Another moist tongue from the south frequently appears over western Florida. This recurrence suggests that the *position* of the eddies very largely must depend upon geographic factors. Nothing is known, however, about the dynamic factors which presumably control the *size* of the eddies.

2. During the last three years we have constructed daily weather maps for the Northern Hemisphere for use in an effort to extend the time range of our synoptic forecasts. In the course of these studies seven-day, later five-day mean pressure charts were constructed weekly. Underlying the construction of these mean charts is the assumption that the averaging process, at least partially, removes perturbations in the pressure distribution associated with the rapidly moving wave cyclones. The remaining perturbations are of larger dimensions and appear to determine the path of the wave disturbances. On the winter maps there are normally at least five such perturbations to be seen, the Icelandic and the Aleutian Lows, the Azores and the Asiatic Highs and finally the Pacific High, but one or several of these centers frequently breaks up into two parts. It is well known that these perturbations at higher levels no longer appear as closed isobaric systems but merely as undulations in the prevailing zonal pressure distribution. The sea level perturbations sometimes move westward several weeks in succession, finally splitting up into several parts. No explanation of these displacements and of the ultimate breaking up of the perturbations has been offered.

3. Five-day mean isentropic charts and five-day mean pressure charts for the three kilometer level have been constructed weekly. These upper level mean pressure charts often indicate the existence, during the winter season, of a trough of low pressure over the United States. The trough may remain stationary for several weeks in succession, but hitherto no adequate explanation of its position and displacement has been offered. The significance of the trough is best seen from the isentropic charts which show that even a fairly feeble low pressure trough over the Mississippi Valley with west-southwesterly gradient wind over the eastern part of the country in isentropic representation appears as a strong moist current extending from the Gulf northeastward towards the middle Atlantic coast.

4. In an attempt to obtain simple indices to the intensity of the general zonal circulation of the Northern Hemisphere, five-day means of the mean pressure on each latitude circle were computed and plotted weekly in the form of pressure profiles from equatorial to polar regions. Such curves were first used systematically by H. H. Clayton (1923). These curves of mean pressure against latitude have on several occasions shown marked trends towards increasing or decreasing zonal circulation, persisting through several weeks.

In the attempt to understand the dynamics of the upper level trough over the United States, the author found great help in a remarkable paper by J. Bjerknes (1937) which offers a simple explanation for the displacement of perturbations superimposed upon the zonal pressure distribution which normally prevails in the upper part of the troposphere. Bjerknes studied the amount of air transported across a section perpendicular to two consecutive isobars and found that this transport depends upon the curvature

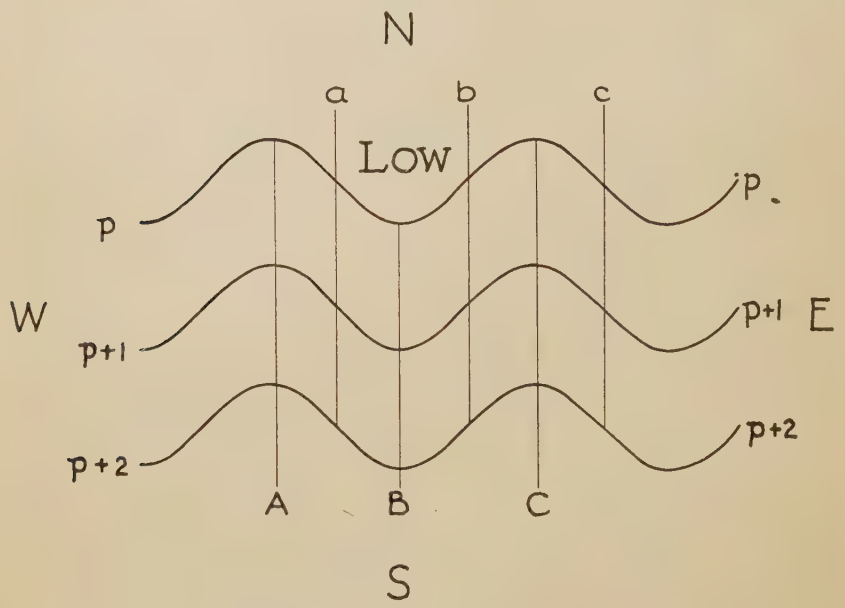

Figure 11. Sinusoidal Perturbation on Zonal Motion.

of the isobars and upon the latitude of the section. Thus, if the pressure distribution is not strictly zonal but characterized by a certain perturbation, regions of convergence and divergence and hence of rising and falling pressure are created which lead to a displacement of the perturbation. Bjerknes' reasoning may be seen from Figure 11. Disregarding at first variations in latitude, it is evident that more air is transported through the sections *A* and *C* where the curvature is anticyclonic than through section *B*, where the curvature of the isobars is cyclonic. Thus horizontal divergence is created between *B* and *C* and a drop in pressure occurs to the east of the trough at *B*, while horizontal convergence and a rise in pressure will occur between *A* and *B*. As a result the trough will move eastward.

Since the variations in the transport and thus in the magnitude of the pressure changes increase with decreasing radius of curvature of the isobars, it follows that for the same amplitude short waves must move more rapidly eastward than long waves.

Considering now solely the effect of variation in latitude, it may be seen from the gradient wind relationship that more air is transported between consecutive isobars in low latitudes than in high. Thus the transport across B is greater than the transport across A or C and hence there must be convergence and pressure rise between B and C, divergence and pressure fall between A and B. The variations in transport are independent of the wave length of the perturbation, but increase with its amplitude. In the absence of other factors, the latitude effect here described would lead to a westward displacement of the perturbations.

For perturbations of the dimensions actually observed in the free atmosphere the two effects are approximately of the same magnitude. It follows that long waves must travel westward and shorter waves eastward. An intermediate wave length must exist such that the corresponding perturbations remain stationary.

The stream line pattern to which the above analysis by Bjerknes applies must satisfy not only the equation of continuity but also the equations of motion. If one considers the very simplest case of an ideal (non-friction) homogeneous, incompressible atmosphere *in purely horizontal motion*, the two equations of motion may, through the elimination of pressure, be compressed into a single equation expressing the conservation of absolute vorticity. Thus, if f represents the Coriolis' parameter ($f = 2\Omega \sin \varphi$) and consequently also the vertical component of the vorticity due to the rotation of the earth, and if ζ is the vertical component of the vorticity of the motion of the air relative to the earth's surface, then each individual vertical column must satisfy the condition

$$f + \zeta = \text{constant}. \tag{1}$$

In this equation

$$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}. \tag{2}$$

It is assumed that the x-axis points eastward, the y-axis northward. Vorticity is counted positive for cyclonic rotation, negative for anticyclonic rotation. Thus equation (1) implies that an air column which is displaced towards higher latitudes, where the cyclonic vertical component of the earth's rotation is stronger, will experience a decreasing cyclonic, or increasing anticyclonic, rotation. A column displaced towards lower latitudes will experience an increasing cyclonic, or decreasing anticyclonic, rotation.

It follows from (1) that the relative vorticity ζ must remain constant in the absence of variations in latitude. The stream lines in Figure 11 show cyclonic vorticity between a and b, anticyclonic vorticity between b and c and hence such a pattern cannot be established through purely horizontal motion except possibly as the result of latitude variations. It is the purpose of the simple analysis below to show that the latitude variations are sufficient to bring about the required vorticity variations.

It follows from (1) that the individual variation of vorticity with time must obey the law

$$\frac{d\zeta}{dt} = -\frac{df}{dt}. \tag{3}$$

Since the Coriolis' parameter does not depend on longitude or time, it follows that

$$\frac{df}{dt} = v\frac{\partial f}{\partial y} = \beta v \left(\beta = \frac{\partial f}{\partial y}\right) \tag{4}$$

and hence

$$\frac{d\zeta}{dt} = -\beta v. \tag{5}$$

In (5) β represents the rate at which the Coriolis' parameter increases northward. It is easily seen that β may be computed from the equation

$$\beta = \frac{\partial f}{\partial y} = \frac{\partial f}{R\partial\varphi} = \frac{2\Omega\cos\varphi}{R}, \tag{6}$$

R being the mean radius of the earth. Table I gives the results of such a computation. In the subsequent analysis it is necessary to treat β as a constant and hence independent of y. This assumption is reasonably justified near the equator where β has its maximum and its rate of variation in per cent is very small, but represents a rather severe restriction in higher latitudes. Thus β decreases by about 29% from 45° N. to 60° N.

TABLE I

VARIATION OF β WITH LATITUDE

φ	$10^{13}\cdot\beta = \frac{2\Omega\cos\varphi}{R}\cdot 10^{13}$
90°	0.0 $cm^{-1}\,sec^{-1}$
75°	0.593
60°	1.145
45°	1.619
30°	1.983
15°	2.212
0°	2.290

We shall now apply (5) to the case of a zonal current of uniform constant velocity U upon which is superimposed a perturbation with the velocity components u' and v'. Thus the total velocities are

$$u = u' + U, \quad v = v'. \tag{7}$$

Since the zonal current U is independent of x and y it follows that

$$\zeta = \zeta' = \frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}. \tag{8}$$

The vorticity equation (5) may be expanded and gives then

$$\frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} = -\beta v \tag{9}$$

or

$$\frac{\partial \zeta}{\partial t} + U\frac{\partial \zeta}{\partial x} + \left[u'\frac{\partial \zeta}{\partial x} + v'\frac{\partial \zeta}{\partial y}\right] = -\beta v'. \tag{10}$$

For small perturbations the terms within the bracket may be neglected as small terms of second order (ζ, u' and v' are all small terms of the first order). Thus the equation reduces to

$$\frac{\partial \zeta}{\partial t} + U\frac{\partial \zeta}{\partial x} = -\beta v'. \tag{11}$$

Now let us look for perturbations traveling without change in shape. If c is the eastward speed of such a wave, it follows that

$$\frac{\partial \zeta}{\partial t} = -c\frac{\partial \zeta}{\partial x} \tag{12}$$

and consequently (11) transforms into

$$(U - c)\frac{\partial \zeta}{\partial x} = -\beta v'. \tag{13}$$

In the case of perturbations independent of y it follows from (8) that

$$\zeta = \frac{\partial v'}{\partial x} \tag{14}$$

and thus

$$(U - c)\frac{\partial^2 v'}{\partial x^2} = -\beta v'. \tag{15}$$

In the case of a simple sinusoidal disturbance of the wave length L the perturbation must be given by an expression of the form

$$v' \backsimeq \sin \frac{2\pi}{L}(x - et). \tag{16}$$

Substitution of (16) in (15) shows that

$$c = U - \frac{\beta L^2}{4\pi^2}, \tag{17}$$

This expression gives the wave velocity in terms of the gradient wind velocity and the wave length L.

It appears that the waves become stationary when

$$c = U - \frac{\beta L_s^2}{4\pi^2} = 0, \quad L_s = 2\pi\sqrt{\frac{U}{\beta}}. \tag{18}$$

Waves of greater length than L_s travel westward (c negative), shorter waves travel eastward. A combination of (17) and (18) gives

$$c = U\left(1 - \frac{L^2}{L_s^2}\right), \tag{19}$$

from which the relation between wave length and speed is most readily seen. The stationary wave length is given in Table II as a function of the latitude and of the zonal circulation U.

TABLE II

STATIONARY WAVE LENGTH IN KM AS FUNCTION OF ZONAL VELOCITY (U) AND LATITUDE (φ)

φ \ U	*4 m/sec*	*8 m/sec*	*12 m/sec*	*16 m/sec*	*20 m/sec*
30°	2822 km	3990 km	4888 km	5644 km	6310 km
45°	3120	4412	5405	6241	6978
60°	3713	5252	6432	7428	8304

The total number of waves (n) around the circumference of the earth at the latitude φ is given by

$$nL = 2\pi R \cos \varphi. \tag{20}$$

Substitution of (20) in (17) gives

$$U - c = \frac{2\Omega R}{n^2} \cos^3 \varphi \tag{21}$$

or, for stationary waves,

$$U = \frac{2\Omega R}{n_s^2} \cos^3 \varphi. \tag{22}$$

The velocity deficit $U - c$ obtained from (21) is given in Table III as a function of the number of perturbations and of the latitude. It is evident that if the zonal velocity distribution is known equation (22) enables one to determine the number of stationary perturbations possible in each latitude.

TABLE III

VELOCITY DEFICIT $(U-c)$ AS FUNCTION OF NUMBER OF PERTURBATIONS (n) AND LATITUDE (φ)

φ \ n	2	3	4	5	6	7
30°	150.7 m/sec	67.0	37.7	24.1	16.7	12.8
45°	82.0	36.5	20.5	13.1	9.1	6.7
60°	29.0	12.9	7.3	4.6	3.2	2.4

The preceding analysis applies to simple harmonic waves. The case of an arbitrary solitary perturbation observed at the time $t = o$ is somewhat more complicated but may be treated with the aid of Fourier integrals. Thus, in the case of a symmetric solitary pressure ridge or trough centered around $x = o$, one finds

$$v' = \int_0^\infty F(\mu) \sin \mu \left[x - Ut + \frac{\beta}{\mu^2} t \right] d\mu, \tag{23}$$

provided

$$v_o' = \int_0^\infty F(\mu) \sin \mu x \cdot d\mu \tag{24}$$

represents the velocity distribution in the initial perturbation (v_o' is antisymmetric). It follows that the harmonics corresponding to large wave lengths $\left(\mu = \frac{2\pi}{L} < \frac{2\pi}{L_s}\right)$ must travel westward, those of shorter wave length $(L < L_s)$ eastward, and thus a large initial perturbation may actually split in two centers, traveling in opposite directions.

V. Bjerknes and collaborators have computed the zonal velocity distribution from the zonal temperature distribution and from the zonal pressure distribution at the ground (V. Bjerknes and coll., 1933). The computation was made separately for the months of February and August. The aerological data used in this computation were, unfortunately, not entirely satisfactory for the purpose, since soundings from stations in widely different longitudes had to be combined (Pavia, Agra, Batavia). Bjerknes' diagram shows that the maximum velocity normally occurs just below the tropopause. The maximum velocities for the month of February have been taken from this diagram and are listed below in Table IV. Intermediate

TABLE IV

MAXIMUM ZONAL VELOCITY IN FEBRUARY ACCORDING TO BJERKNES

Lat.	*70°*	*60°*	*53°*	*45°*	*30°*	*28°*
U_{max} (mps)	10	[16]	20	[23]	[29]	30

values have been determined by interpolation and are enclosed within brackets. A study of the tabulated values indicates that this portion of the atmosphere rotates with a practically constant relative angular velocity. A comparison with Table III shows that the number of stationary perturbations must be between two and three in 60° N. and slightly above four in latitude 30° N. This result is in fair agreement with the observed dimensions both of the permanent lows and of the subtropical high pressure cells.

In applying the preceding analysis to the semipermanent centers of action, it must of course be emphasized that these centers are maintained in their normal position by permanent solenoidal fields acting alone or in cooperation with topographic factors. Thus there is no obvious reason why the character of these thermally or topographically produced "forced" perturbations should agree in detail with the stationary "free" perturbations analyzed above. To the extent that these forced perturbations are the result of solenoidal circulation they are controlled primarily by the distribution of solenoids in a vertical surface following a latitude circle around the earth. In such a surface the solenoids will be found to have their maximum concentration wherever the latitude circle crosses the boundary between continent and ocean. (Bjerknes, V. and coll., 1933, pp. 686–693). During the warm season and in lower latitudes the concentration of solenoids is particularly marked along the west coasts of North America and of Europe-Africa, during the cold season and in lower latitudes along the Pacific coast of Asia and over the eastern coast of North America. During either season the two principal regions of solenoidal activity are much further apart than the "free" stationary wave length determined above. Each one of these solenoidal zones will create a practically stationary perturbation in the zonal circulation and in the trail of this perturbation a series of stationary waves must develop in somewhat the same fashion as the standing waves which are sometimes observed in the clouds on the lee side of a mountain ridge. Mathematically the problem may be expressed by saying that if, through solenoidal or topographic influences, a perturbation of the zonal circulation is maintained at the line $x = o$, such that the perturbation velocity and vorticity there have arbitrarily prescribed values,

$$v'_{x=o} = v_o', \; \zeta'_{x=o} = \left(\frac{\partial v'}{\partial x}\right)_{x=0} = \zeta_o', \tag{25}$$

then it follows from (15) that

$$v' = v_o' \cos \frac{2\pi x}{L_s} + \frac{\zeta_o' L_s}{2\pi} \sin \frac{2\pi x}{L_s}, \tag{26}$$

for all values of $x > o$. This solution corresponds to a series of stationary perturbations of the wave length L_s. It is the author's opinion that this reasoning may help to explain the fact that the Pacific high pressure belt during the winter season generally splits into two separate cells. It is probable that upper level troughs formed in this fashion readily develop into frontogenetic zones near sea level.

TABLE V

COMPONENT FROM THE WEST (IN M.P.S.) OF VECTORIAL MEAN WIND

Va Winter

Station	*Lat. N.*	*6 km.*	*8 km.*	*10 km.*	*12 km.*	*14 km.*
Ellendale	46.0	11.2	10.3	10.7	—	—
Omaha	41.2	10.7	8.8	8.6	—	—
Broken Arrow	36.0	14.3	12.6	13.7	20.3	—
Groesbeck	31.5	13.9	12.8	14.2	17.8	—

Vb Summer

Ellendale	46.0	10.2	11.9	11.2	11.1	—
Omaha	41.2	6.3	7.4	6.1	9.6	11.5
Broken Arrow	36.0	2.8	3.1	3.3	4.0	3.8
Groesbeck	31.5	0	−0.5	−0.5	0.9	0.5

Bjerknes' summer section is, unfortunately, very much disturbed by the Indian monsoon, and must therefore be used with a great deal of caution. It shows a velocity maximum of about 30 mps. in latitude 40° N. and would thus correspond to about four stationary high pressure cells in this latitude. However, the observed zonal velocity distribution over the United States differs rather markedly from the one computed by Bjerknes. In Tables Va and Vb are given the components from the west of the vectorial mean wind velocity at different heights above four stations in the vicinity of the meridian of Omaha, Nebraska, for winter and summer. They were determined graphically from vectorial mean winds published by the U. S. Weather Bureau (Stevens, L. A., 1937). These velocities are considerably less than those computed from the zonal temperature distribution. The tables show that during the winter season the west wind velocity is around 12 mps. up to about 12 km., whence it increases rapidly upward. In summer time the wind distribution is characterized by a strong anticyclonic shear in the

north and by weak velocities over the greater part of the country. Assigning a value of 16 mps. to the zonal circulation during the winter season one finds, from Table III, that the number of permanent perturbations would be about 6 at 30° N., and between 4 and 5 at 45° N. Unfortunately there are no pilot balloon data available concerning the wind distribution at higher latitudes in the same meridian. If Bjerknes' computed value of the zonal circulation at latitude 60° N. is combined with the observed wind data at lower latitudes, the resulting zonal circulation is compatible with the number of perturbations observed in the polar regions. The slightly larger number, i. e. slightly smaller dimensions, of the subtropical cells is not incompatible with the actual dimensions of the anticyclones of lower latitudes.

Table Vb indicates that the summer season is characterized by a very weak zonal circulation over the United States with strong anticyclonic shear zone near the Canadian border. If a value of 4 mps. is assigned to the zonal wind, one obtains from Table II a stationary wave length of about 3000 km., in good agreement with the dimensions of the anticyclonic cells observed on our mean isentropic charts for the summer season.

The barotropic perturbations analyzed above are not associated with any available energy supply and are therefore stable and incapable of changing into real vortices. However, it is well known that a shear zone such as the one observed in the northern part of the country during the summer season is dynamically unstable and must break up into vortices which convert the energy of the zonal current into vortical kinetic energy. In this case, the equation for the conservation of vorticity should be modified to include the vorticity of the zonal current. Thus if the vorticity of the zonal current is designated by Z it follows that equation (13) changes into

$$(U - c)\frac{\partial \zeta'}{\partial x} + v'\frac{\partial Z}{\partial y} = -\beta v' \tag{27}$$

or, since

$$Z = -\frac{\partial U}{\partial y}, \tag{28}$$

$$(U - c)\frac{\partial \zeta'}{\partial x} = -\left(\beta - \frac{\partial^2 U}{\partial y^2}\right)v'. \tag{29}$$

An inspection of Table Vb shows that $\frac{\partial^2 U}{\partial y^2}$ at the most is about 25% of β. Since $\frac{\partial^2 U}{\partial y^2}$ is positive, the stationary wave length is thus increased by a maximum of about 13%. It should be remembered, however, that the original solution no longer strictly holds, since the zonal velocity (U) is a function of y and the perturbation thus also must depend on that coordinate.

The solenoidal field along the western coast of North America in combination with the steep mountain ranges creates a permanent perturbation (trough) near the coast and the permanent perturbation thus maintained will set up a series of standing perturbations in the zonal pressure distribution further down stream. The shearing zone along the northern border of the United States must necessarily break up into eddies which take care of the dissipation of the kinetic energy generated further to the north. It seems likely that the maximum size to which the frictionally driven eddies can grow is determined by the grid provided by these semi-permanent perturbations.

This grid would seem to explain the appearance of a moist tongue over western Florida in the isentropic mean charts for August and suggests an explanation for the occasional occurrence of a third moist tongue in the vicinity of Bermuda.

The large semi-permanent Bermuda high pressure cell observed on the seasonal mean sea level pressure charts, on the north side of which this grid of smaller perturbations is superimposed, is not explained by the preceding analysis but appears to require a permanent and widespread field of solenoids for its maintenance. It is also possible that the larger dimensions of this system are associated with the higher zonal wind velocities prevailing near the base of the stratosphere.

The simple analysis carried out above does not include the effect of changes in depth of the moving fluid column. A single layer atmosphere, moving with a constant zonal velocity U must have a depth D_o which decreases northward. This slope gives rise to a pressure gradient which balances the Coriolis' force associated with the zonal motion. Thus

$$U = -\frac{g}{f}\frac{\partial D_o}{\partial y}. \tag{30}$$

It follows that fluid columns moving north or south are going to change their depth and as a result their vorticity will change. In addition, the traveling perturbations themselves cause deformations of the free surface which in turn produce changes in vorticity. To analyze the problem completely we must return to the general equations of motion. For a homogeneous incompressible atmosphere of thickness D they may be compressed into the following system:

$$\frac{d\zeta}{dt} = -\frac{df}{dt} - (f+\zeta)\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \tag{31}$$

$$\tfrac{1}{2}\frac{dq^2}{dt} = -g\left(\frac{dD}{dt} - \frac{\partial D}{\partial t}\right) \tag{32}$$

$$\frac{1}{D}\frac{dD}{dt} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right). \tag{33}$$

The first of these equations expresses the conservation of vortices, the second the conservation of energy, and the third the conservation of mass. In these equations q^2 is the kinetic energy per unit mass and g the acceleration of gravity.

In our particular case

$$u = U + u',\ v = v',\ q^2 = U^2 + 2u'U + (u'^2 + v'^2) \tag{34}$$

and

$$D = D_o + \delta \tag{35}$$

D_o being the depth (decreasing northward) of the undisturbed atmosphere in zonal motion and δ the deformation due to the travelling perturbations. Neglecting squares and products of u' and v' one finds

$$\tfrac{1}{2}\frac{dq^2}{dt} = U\frac{du'}{dt} = U\left(\frac{\partial u'}{\partial t} + U\frac{\partial u'}{\partial x}\right) \tag{36}$$

and

$$\frac{dD}{dt} = \frac{\partial \delta}{\partial t} + U\frac{\partial \delta}{\partial y} + v'\frac{\partial D_o}{\partial y} \tag{37}$$

or, because of (30),

$$\frac{dD}{dt} = \frac{\partial \delta}{\partial t} + U\frac{\partial \delta}{\partial x} - \frac{fU}{g}v'. \tag{38}$$

If we replace $f + \zeta$ on the right hand side of (31) with a constant f_o* and introduce a constant mean value D_∞ for the undifferentiated D in equation (33), equations (31) to (33) are readily solved. For perturbations which are independent of the y-coordinate, one obtains finally an equation for the wave speed which may be written

$$(U - c)\frac{4\pi^2}{L^2} = \beta + \frac{cf_o^2}{gD_\infty - (U - c)^2}. \tag{39}$$

It is interesting to note that for $U = 0$, $\beta = 0$, this equation gives the wave velocity obtained by Sverdrup (1926) in his study of tidal waves on the North Siberian shelf. For $U = 0$, $\beta = 0$, $f_o = 0$ it reduces to the well known formula for long gravitational waves

* This substitution is justified in view of the small contribution of the divergence term in (31) to the individual change in vorticity in a single-layer barotropic atmosphere.

$$c = \pm \sqrt{gD_\infty}. \tag{40}$$

For standing perturbations ($c = 0$) it reduces to the relation obtained earlier in this paper by elementary methods,

$$L_s = 2\pi \sqrt{\frac{U}{\beta}}. \tag{41}$$

For a homogeneous atmosphere the product gD_∞ has a value of about $8 \cdot 10^8 \cdot \text{cm}^2 \cdot \text{sec}^{-2}$. For the perturbations in which we are interested $(U - c)^2$ is of the order of magnitude $10^7 \cdot \text{cm}^2 \cdot \text{sec}^{-2}$ or less. Thus it is a permissible approximation to write

$$(U - c)\frac{4\pi^2}{L^2} = \beta + \frac{cf_o^2}{gD_\infty}, \tag{42}$$

or, after solving for c,

$$c = \frac{U - \dfrac{\beta L^2}{4\pi^2}}{1 + \dfrac{L^2}{4\pi^2\lambda^2}}, \tag{43}$$

in which expression the length λ, defined by

$$\lambda = \frac{1}{f_o}\sqrt{gD_\infty}, \tag{44}$$

has a value of about 2800 km. for a homogeneous atmosphere. Even for wave lengths double the value of λ the wave velocity determined by (43) is only 10% less than the one obtained from the approximate formula (17).

This formula has an important forecasting implication. If a series of permanent perturbations exists at the time $t = 0$ and a study of consecutive pressure profiles suggests that the zonal circulation is slowing down, it follows that the perturbations will be displaced westward, but the rate of this displacement must be less than the reduction in the zonal circulation velocity.* It is likewise evident that an acceleration of the zonal circulation speed will bring about an eastward displacement of the perturbations, but again at a slower rate than the change in the zonal velocity.

* A preliminary analysis of perturbations in the upper, moving portion of a double-layer atmosphere with a bottom layer at rest indicates that the denominator in (43) then may reach values of 4 or 5; thus the rate of displacement of non-stationary perturbations is greatly reduced. However, the final answer to this question cannot be given until a satisfactory theoretical analysis has been carried out. The same preliminary result applies to a double-layer atmosphere in which the motion occurs in the lower layer.

It was stated earlier than an arbitrary perturbation, once created, will be displaced in such a fashion that harmonics of greater wave length will move westward, those of shorter wave length eastward, and that this process may lead to a splitting up of the perturbation. It is now evident that such displacements may be produced also by changes in the zonal circulation. It is evident that once the centers of action have been displaced from their normal positions, the fairly stationary solenoidal fields responsible for their generation and maintenance may lead to a regeneration of the centers in their normal position. In this way double centers will again be produced. An inspection of our northern hemisphere charts appears to indicate that this is particularly likely to happen when the Asiatic High, as a result of a

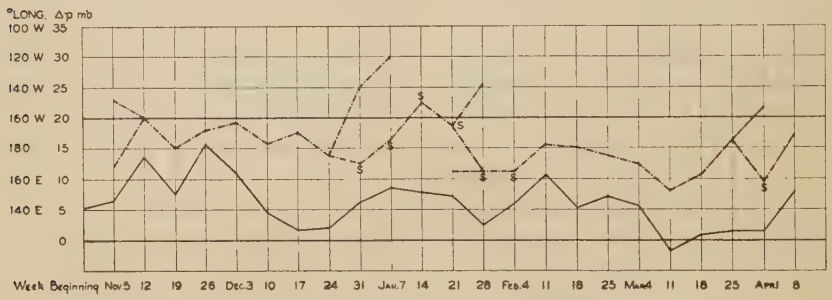

Figure 12. Relation between position of Aleutian Low and Zonal Circulation Intensity. Broken Line Indicates Longitude of Aleutian Low, as Determined from our Weekly Five Day Mean Pressure Charts and Full Line gives Zonal Circulation Intensity as Indicated by Mean Pressure Difference on the same Charts between 35° N. and 55° N.

slowing down of the general zonal circulation, has been displaced far to the west (towards Europe) of its normal position.

Two practical examples to illustrate the preceding theoretical analysis are presented below. Figure 12 contains a comparison for the winter 1938–1939 between the longitude of the Aleutian Low, as determined by my colleague Dr. H. C. Willett from the five day mean pressure charts prepared weekly under his direction at the Massachusetts Institute of Technology, and the intensity of the zonal circulation as measured by the pressure difference between latitudes 35° N. and 55° N., obtained by averaging this pressure difference all around the globe on the same charts. There are a few cases of splitting of the center and in these cases double points have been entered. If in each case the stronger of the two centers (marked *s*) is selected, it is readily seen that a good correlation is obtained between the two curves. There is also some evidence of a few days lag of the longitude curve relative to the circulation index. The correlation coefficient between the two curves is 0.665. A later and more accurate determination of the

zonal circulation index resulted in a correlation coefficient of 0.695. Finally, to prove that the variations in the intensity of the zonal circulation index are not merely the result of the longitude variations of the Aleutian Low, a revised zonal circulation index was computed which excluded a section of 120 degrees of longitude over the Pacific. Even then a correlation of 0.542 with the longitude of the Aleutian Low was obtained.

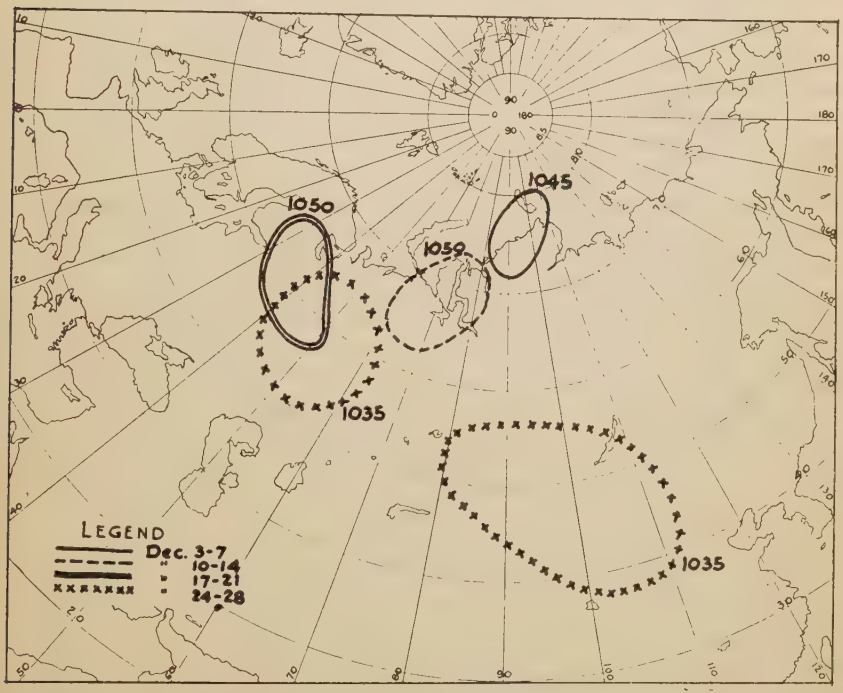

Figure 13. Successive positions of last closed isobar in Asiatic High during December 1938. These isobars were obtained from our weekly five day mean pressure charts.

It may at first sight appear unreasonable to compare the displacement of the Aleutian Low with the zonal circulation intensity in a much lower latitude. It must be remembered, however, that the positions and displacements of the semipermanent centers of action depend upon the intensity of the zonal motion in the free atmosphere well above the seat of the advective phenomena. The author has tried to demonstrate, in a previous paper (Rossby, 1938), that at least during the summer season, the westerlies south of the polar front are to a large extent maintained by isentropic stresses which transmit the motion of the polar vortex to the surface layers

in somewhat lower latitudes. On the basis of this analysis, it appears reasonable to measure the intensity of the polar vortex in the free atmosphere by the sea level pressure gradient to the south of the polar front.

Figure 13 shows the successive positions of the innermost closed isobar of the Siberian High during a few weeks in December, 1938, as determined from the weekly five day mean pressure charts for this period. An inspection of the circulation index in Figure 12 shows that this period was characterized by a continuous decrease in the zonal circulation intensity, and Figure 13 shows that the Siberian High during this period advanced towards northwestern Europe, where its arrival caused a complete change in weather type. This example is particularly significant, since the High is located in entirely different latitudes from the ones used in determining the circulation intensity. It is further more located over the interior of the continent where the normal zonal pressure distribution is no longer discernible.

It is perhaps not superfluous to emphasize that the zonal circulation intensity varies with the intensity of the solenoids in meridional planes, while the maintenance of permanent perturbations on this zonal circulation primarily must be the result of solenoids contained in vertical planes parallel to the latitude circles.

Further examples of the relation between variations in the zonal circulation intensity and displacements of the atmospheric centers of action are discussed by Dr. Willett in another place (Willett 1939).

ACKNOWLEDGEMENT

The author is greatly indebted to his collaborators and colleagues, Messrs. Allen, Holmboe, Namias, Page and Willett for whole-hearted cooperation and many valuable suggestions, based on their intimate contact with the synoptic and statistical aspects of the problem of long range weather forecasting.

REFERENCES

Bjerknes, J.
1937. Die Theorie der aussertropischen Zyklonenbildung, *Meteorologische Zeitschrift*, Vol. 54, pp. 460–466.

Bjerknes, V. and collaborators.
1933. *Physikalische Hydrodynamik*, Julius Springer, pp. 648–653.

Clayton, H. H.
1923. World Weather, The MacMillan Co., New York.

Ekman, V. W.
1932. Studien zur Dynamik der Meeresströmungen, *Gerlands Beiträge zur Geophysik*, Bd. 36, pp. 385–438.

Namias, J. and Wexler, H.
1938. Mean Monthly Isentropic Charts and their Relation to Departures of Summer Rainfall, *Transactions of the American Geophysical Union*, Nineteenth Annual Meeting.

Rossby, C.-G.
1938. On the Maintenance of the Westerlies South of the Polar Front, in *Papers in Physical Oceanography and Meteorology*, Vol. VII, No. 1.

Sverdrup, H. U.
1926. Dynamics of Tides on the North Siberian Shelf, *Geofysiske Publikasjoner*, Vol. IV, No. 5.

Stevens, L. A.
1937. Winds in the Upper Troposphere and Lower Stratosphere over the United States, *Monthly Weather Review*, Supplement 36.

Willett, H. C.
1939. Certain Statistical Relationships Bearing on the Preparation of Five-Day Weather Forecasting in the United States, presented at the April 1939 meeting of the American Meteorological Society in Washington, D. C., and to be published later on.

CORRELATIONS IN AQUATIC ECOLOGY

WITH AN EXAMPLE OF THEIR APPLICATION TO PROBLEMS OF PLANKTON PRODUCTIVITY

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Much of the ecological work of the present time is concerned with the direct observation of natural variations in environment and biota, followed by an examination of the relationships between variables and speculation as to their meaning. This is the most efficient method of attaining a general viewpoint of any ecological situation, and to use it as a preliminary step to more detailed experimental analysis is economical of time and effort. It is therefore important for the ecologist to have a clear understanding of the methods of analyzing variations and of using the results in the construction of biological theories.

In many cases variations can be analyzed merely by inspection of the graphs or numerical data. Needless to say, the greater part of the advances in our knowledge of ecology have been made by this method. All the more important relationships are revealed by this means, and in the hands of a careful and experienced worker, the results are semi-quantitative.

It should be pointed out, however, that all scientific generalizations are statistical generalizations. Unless the relationship is absolute, so that the data fit perfectly, there is a certain probability that the observed relationship is due only to chance variation. To generalize by inspecting the graphs or numerical data is to make a guess that this probability is insignificantly small. The statistical technique is more exactly quantitative, for not only does it permit an accurate mathematical statement of the probability, but also it provides a coefficient of correlation which is numerically accurate within the limits of its probable error, so that it is possible to compare the relative degrees of relationship between the different sets of variables. In view of these advantages, it seems unfortunate that ecologists have not made more use of the methods of analyzing inter-related variants, particularly since they have proved to be extremely valuable in such related fields as agriculture, forestry, and growth studies in general.

In recent papers the writer has used regression equations and correlations of various sorts as an aid in the explanation of ecological data. The results appear to justify the application of these techniques, and they are being used extensively for work on marine plankton now in progress. But as certain problems have arisen which demonstrate that the proper use of

correlations requires considerable care, and that they can be a hindrance rather than a help if used unwisely, it seems desirable at this time to discuss the theoretical principles that are involved. Mathematical thought serves as a convenient intermediate step between the raw data and the completed biological concept, but it is necessary to effect a synthesis of statistical and biological theory in order to weigh the results fairly. This paper is therefore an attempt to outline in an elementary way the rules that govern the use of correlations in ecological work. It is what might be termed a study of the relationship between statistical and biological significance.

The latter term is used in a restricted sense, which may be explained by the following definitions:

Quantitative biological significance—The significance of a factor, which may or may not be essential to life, the variations of which impose related variations on the quantity of life occurring with it.

Qualitative biological significance—The significance of a factor which is essential to life, or excludes life, but does not by its variations cause related variations in the biocoenosis.

Obviously, the correlation technique is capable of dealing only with quantitative biological significance. It should be pointed out, however, that the distinction is real only as applied to a particular biological association. For example, in a typical marine environment the gross salts have only qualitative significance. But in comparing different environments with a wide range of salinities, the variations of the latter might have considerable effect on the quantity of a given species living in the various environments, and the relationship could be evaluated quantitatively by means of correlations.

If the relation between the environmental factor and the living organism is accurately described, the degree of biological significance is naturally proportional to the degree of mathematical significance. The simplest measure of the latter is the coefficient of correlation. It will be shown later, however, that in ecological studies, the constant of the regression equation is generally a more useful value. This constant is derived from the correlation, the standard deviations of the variables, and, in the case of non-linear regressions, such additional constants as are necessary to describe the form of the relationship.

The use of correlations implies two beliefs which, from the philosophical point of view, are assumptions, but to the practical biologist are reasonable according to the facts available. In the first place, it is assumed that the pairs of numbers have a biological meaning, that, within the limits of technical error, they are true statements of the relative number of units of some organism or environmental element. Having given the numbers a biological meaning, the question arises as to whether the correlation has a biological significance. It has if we assume that such a relationship indi-

cates a causal connection. Just what causes and effects are involved can be determined only by examining all the other facts that bear on the problem. If it is possible to analyze all the inter-related factors in such a way as to reduce the correlation to an expression of a single cause and a single effect, it becomes an accurate index of the biological significance of the relationship. If this is not done, there remains a certain biological significance, for it is the statistical summation of all the factors involved. Whether or not it is an accurate statement of the relationship that it is supposed to represent depends entirely on the relative importance of the indirect relationships that have not been discovered.

It is not intended that these qualifications should set an impossibly high standard for the use of correlations in ecology. The biologist should realize, however, that the relations on which he founds his concepts are a second order of factual material and at best are no more perfect than the original data. If a theory suggests a certain relationship between two sets of variables, a significant correlation lends support to the theory in so far as it shows that the facts are not inconsistent with the theory. At the same time, it does not deny the plausibility of an alternate theory, which would explain the relationship in some other way. It follows that a correlation neither propounds nor proves a theory. The correlation is merely a fact, to be weighed in relation to the other facts. Since all the facts are not at hand, the scientific philosophy demands that the result be regarded as an approximation, to be further improved by subsequent investigations.

There are two general classes of ecological problems that are well suited to investigation by the correlation method. The first is the purely ecological study of a given biological association in respect to the effects of physical, chemical, and biological factors. It has been used by the writer for the investigation of plankton associations, and appears to be practicable for work on other types of associations as well.

The second class of problems is that of the environmental aspects of geographical distribution. A problem of this sort would compare the relative number of individuals of a given species in various localities with the geographical range of factors that might be of importance, such as maximum and minimum temperature and salinity, food supply, number of competitors and predators, etc. While there is little opportunity for extensive application of this method at the present time because of the lack of sufficient data, it appears to be a profitable method for future work, particularly in cases where the distribution of a species is dependent on the interaction of a number of factors. It would be important not only for investigating the relative influence of a given factor in different regions, but also for evaluating the effects of factors such as the depth of water, that are important biologically but cannot be studied quantitatively in any one locality because of their constancy.

The sections that follow will deal with some of the specific problems that arise in the application of correlations to ecological work, and the methods for eliminating errors.

INTER-RELATED VARIATIONS

In any ecological investigation it is generally found that all the factors examined are more or less inter-related, that variations in one factor have an influence, either direct or indirect, on all the other factors. It was pointed out above that in order to arrive at an accurate statement of the biological significance of a factor, it is necessary to isolate simple statements of cause and effect from the maze of inter-relationships. Such isolation is effected by developing a theory (on a basis of extrinsic biological probability) to explain the interaction of various phenomena, and then testing the theory by the multiple correlation method. According to the theory, the variations of a certain entity are dependent on the variations of a number of environmental factors, which are theoretically independent of each other. The independent variables may actually be related, but the multiple correlation method makes corrections for such inter-relationships. The result is a multiple regression equation in which the variations of the dependent variable are expressed in terms of the variations in the series of independent variables. The multiple correlation coefficient obtained from the equation does not prove the correctness of the theory, for it implies no causal significance; however, it shows whether or not the facts are consistent with the theory, and it permits the calculation of errors that might be due to undetermined causes.

The details of the method of examining inter-related variations are readily shown by an example from a work by the author now in press on the plankton of Linsley Pond, a small lake in southern Connecticut. During a period of about eight months, from September, 1937 to June, 1938, measurements were made of phyto- and zooplankton, temperature, light, and nutrients (for the latter the author is indebted to Dr. G. E. Hutchinson), and, in addition, an experimental determination was made of the oxygen production and consumption of the plankton. The results of the analyses are shown in Figure 14. The values listed as means for the entire lake were calculated from analyses at five depths, from 0 to 13 meters, with due allowance for change in the volume of water with depth.

The oxygen production was used to determine the gross production of glucose by the phytoplankton. Such an estimate is subject to some error, since the experimental results are probably not exactly the same as what actually occurs in the open lake water. The discussion in the paper mentioned above, of the errors involved, suggests that the experimental estimate may be as much as 30% too low, although this is definitely a maximum

estimate of the error. And since the error appears to be largely systematic, the results are regarded as a useful approximation of the photosynthetic activity of the plankton.

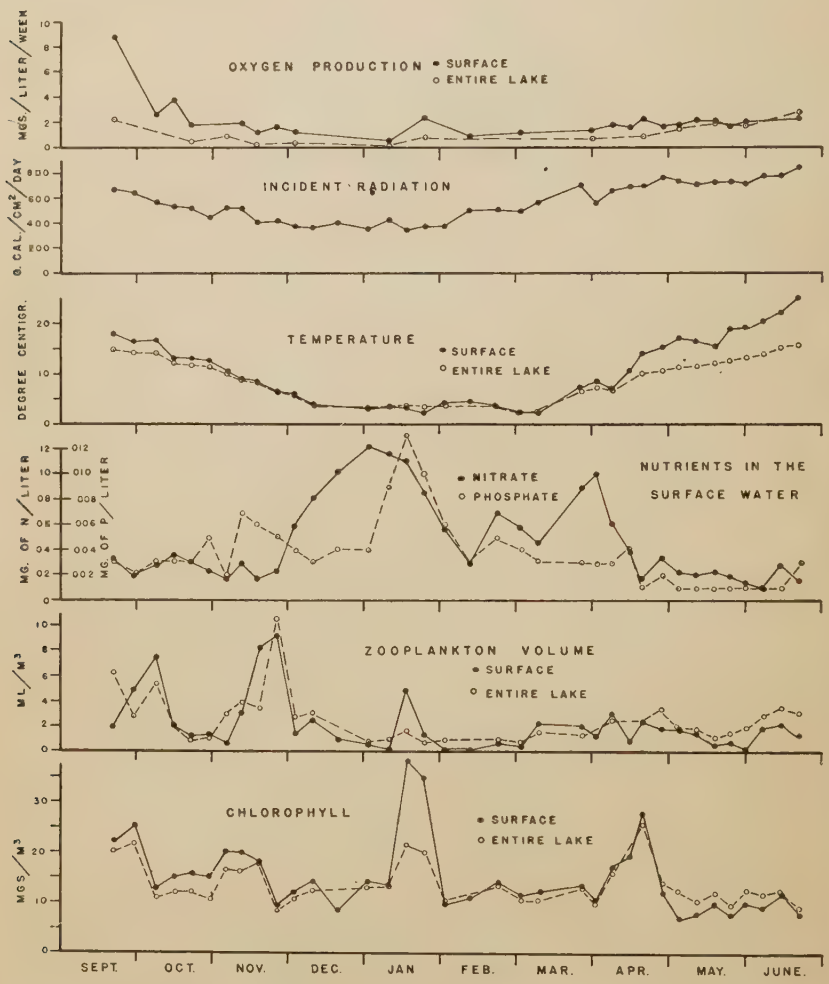

Figure 14. Seasonal variations in Oxygen Production, Radiation, Temperature, Nitrate, Phosphate, Zooplankton and Chlorophyll in Linsley Pond, 1937–38.

At this time it is desirable to extend the analysis further and to attempt an estimate of the phytoplankton production of the lake and its relationship to environmental factors, not only for the intrinsic value that such an

estimate may have, but also *to demonstrate the way in which theoretical generalizations may be derived from a mathematical superstructure.*

Part of the gross production is utilized in the maintenance of the plant body. The remainder is either stored, as carbohydrate or fat, or used in the production of new protoplasmic material. The first step in the estimate of phytoplankton production is therefore to obtain the difference between the total oxygen production and the phytoplankton respiration. This difference may be used as an index of the potential organic production.

To obtain the desired result it is therefore necessary to attempt an estimate of phytoplankton respiration. The experimentally determined values for oxygen consumption are not suitable, because they include the respiration of the other members of the plankton association, the animals and bacteria. An approximation, however, may be made in the following way: The relationship between temperature and the quantity of oxygen consumed per milligram or organic matter in the total seston was plotted, and it was found that the data could be fitted with a logarithmic curve, in which the correlation between temperature and the log oxygen consumption was .735. The phytoplankton respiration was then determined by applying the results obtained from the smoothed curve to the estimated weight of actual phytoplanktonic material. This is an inexact estimate because it assumes that the respiration of all the seston fractions is the same. But although the oxygen consumption per unit weight of zooplankton is greater than that of the plants, the error is at least partly compensated by the fact that some of the seston consists of detritus which is not at the moment undergoing decomposition and therefore is not consuming oxygen. Furthermore, the estimated respiration is so small compared with the oxygen production that if the percentage error were as much as 100, it would not have a more significant effect on the final result than an error of only 10% in the value for oxygen production.

The estimated mean phytoplankton respiration was 0.206 mg. per liter per week at the surface, and in the entire lake it was 0.144 mg. The total range was 0.05 to 0.87 mg. The value for respiration for each date was subtracted from the oxygen production. The remainder was assumed to be proportional (with certain reservations to be discussed below) to the productivity—that part of the photosynthesis that gives rise to the production of new plant material.

The values required for the determination of the effect of environmental factors on this so-called "production" are shown in Tables I and II.

When all the factors are related, as Table II shows them to be, it is impossible to draw any definite conclusions without first making allowance for inter-relationships. For example, the correlation between production in the entire lake and the mean lake temperatures is .793. Since it is known from previous work that temperature has a direct effect on growth rates, we

conclude that in this particular lake the productivity is influenced by variations in temperature.

It would be possible at this point to draw a regression equation to determine what part of the variations in production are directly related with

TABLE I

MEANS AND STANDARD DEVIATIONS

	Surface		*Entire lake*	
	Mean	σ	*Mean*	σ
Production	1.850	1.494	0.921	0.747
Chlorophyll	14.67	6.83	15.00	3.993
Temperature	11.691	5.814	9.758	3.707
Light	581.7	230.8	593.1	167.9
Animals	0.0234	0.0261	0.0198	0.0197
Nitrate	0.0398			
Phosphate	0.0034			

Production, mgs. of oxygen per liter per week; chlorophyll, mgs. per m^3; light, *g.* cal. of incident radiation per cm^2 per day; animals, estimated respiration in mgs. of oxygen per liter per week; nitrate and phosphate, mgs. per liter.

TABLE II

CORRELATIONS

		Production	*Chloro-phyll*	*Tempera-ture*	*Light*
Chlorophyll	Surface	.303			
	Mean	−.151			
Nitrate	Surface	−.392	.199		
Phosphate	Surface	−.140	.570		
Temperature	Surface	.356	−.156		
	Mean	.793	−.115		
Light	Surface	.136	−.235	.723	
	Mean	.700	−.117	.778	
Animals	Surface	.156	−.035	.435	.096
	Mean	.210	.086	.216	.388

temperature. But further examination of the facts shows that it is unwise to do so. Since the amount of solar radiation is correlated with production ($r = .700$), and since variations in temperature and light are also related ($r = .778$), it follows that the biological significance of the temperature-

productivity correlation is not necessarily the same as the mathematical significance, for if light also has a biological importance, the ecological effect of that part of the radiation which varies with temperature will be included in the temperature-production correlation. Statistical procedure offers a simple solution to the difficulty in the form of partial correlations, which make a correction for the relationship between independent variables. In this particular case, the coefficients of partial correlation are as follows: production-temperature, .628; production-light, .213.

When the partial correlations for all the factors are calculated, it is found that only chlorophyll and temperature have a statistically significant effect on production at the surface, and in the entire lake only temperature and light.

The multiple regression equations obtained from the partial correlations and the means and standard deviations of the variables are as follows:

$$W = 0.080X + 0.106t - 0.57 \quad (1)$$

$$W = 0.127t + 0.00095L - 0.87 \quad (2)$$

where W is production in mgs. of oxygen per liter per week, X is chlorophyll in mgs. per m.^{3}, t is temperature, and L is light in g. cal. per cm.^{2} per day. Equation (1) for production at the surface has a multiple correlation coefficient of .533, and for equation (2), the mean production for the entire lake, it is .821.

This is a closer approach to the true biological significance of the variables than the simple correlations were. But the examination is incomplete; there are probably other factors that affect production. Their relations with light and temperature would impose further corrections on the partial correlations. Although the other factors examined did not have statistically significant relationships with production and were therefore omitted from the analysis, this was due in part to the lack of sufficient data, so the correlations were at best only approximations of the biological significance of the factors.

NON-LINEAR REGRESSIONS

The correlations have thus far been assumed to be of the simple, linear type. When the biological relations are not linear, the data must be fitted with the proper curve in order to prevent distortion of the mathematical expression of the relationship.

In the investigation referred to above, there was evidence that some of the factors had a non-linear effect on production. Application of curvilinear methods not only improved the relation of production and temperature, but also it showed that light is a significant factor in the surface waters.

When the equation for productivity at the surface is expanded to include the effects of light, it becomes

$$W = .076X + 0.136t - 0.00106L - 0.24 \quad (3)$$

The final correlation is .488, which is not as good as that obtained from equation (1). The advantage of including the effects of light is apparent, however, on application of the method of Bruce and Reineke (1931) for curvilinear correlations. The technique may be outlined as follows: The equation for the variations of W on X is calculated from (3) by inserting the means of the other independent variables. The same process is repeated for t and L. These regression equations are then drawn, and around them are plotted the residuals—the difference between the observed and the calculated values of W for each observed value of the independent variable. If the points fit the regression line, the variable has a linear function, but if there is a significant departure, the correlation requires a curvilinear technique. An alignment chart is prepared, from which the values for W may be determined for any given set of values for the independent variables. The plotted values are then fitted by the required curve, and the scale of the alignment chart is altered according to the departure of the curve from the linear regression line. This process is repeated until the best possible fit is obtained.

The alignment chart and the regression curves for productivity at the surface are shown in Figure 15. The effect of chlorophyll is linear, and that of temperature is nearly so. The optimum light intensity lies at about 560 g. cal. per cm.^{2} per day and decreases above and below that point. The correlation between the calculated and observed values for productivity is .643.

Application of the curvilinear method to the equation for mean productivity raises the correlation to .926. The regression curve for temperature shows a departure from the calculated regression line that is far more marked than at the surface and appears to have an exponential form. The effect of light remains direct and linear throughout the entire range.

The relationship between the observed and calculated values for productivity is shown in Figure 16. During the greater part of the period of investigation the surface productivity relationship was fairly close. The equation was a failure only during the first month of the study, and the fact that the correlation was less at the surface than in the entire lake was due to the differences observed at that time. There is no reason to believe that there were analytical errors in the first four sets of data. It appears rather that there are special conditions involved in the blooms of the late summer and early autumn which are not accounted for in the statistical treatment.

The effect of light on surface production is a particularly marked example of the error resulting when a curvilinear relation is not treated as

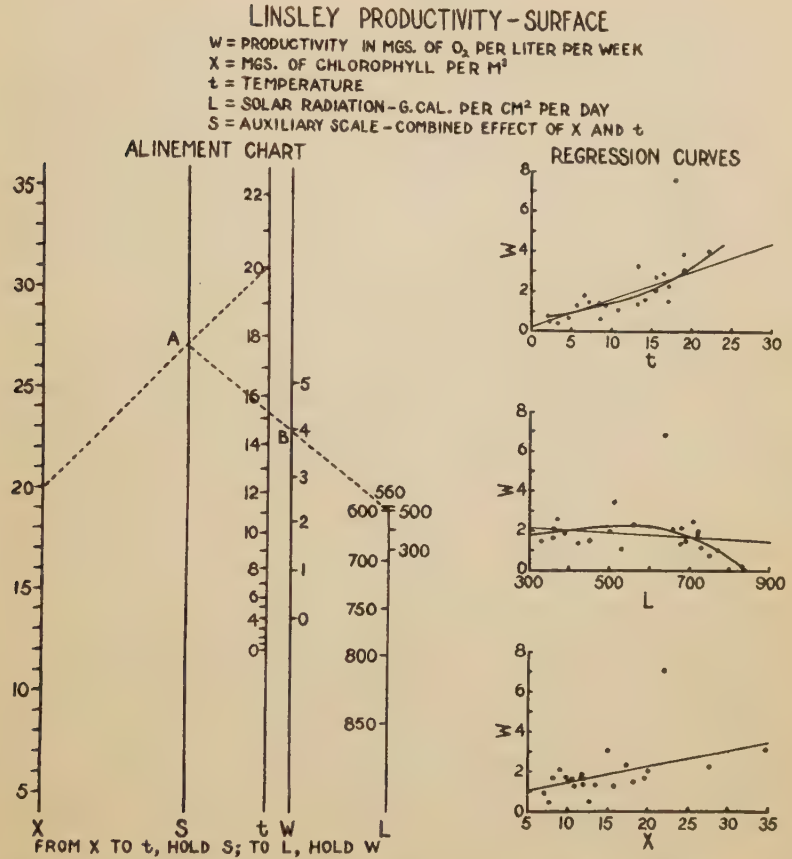

Figure 15. The alignment chart was calculated from the regression equation in such a way that the values for production could be determined for any given set of values for chlorophyll, temperature, and light. The scales on the chart, originally linear, were altered in accordance with the form of the regression curves shown on the right, which fit the data better than the original linear regressions. The alignment chart is used in the following way: To determine the production when the temperature is 20°, the chlorophyll is 20 mgs. per $m.^3$, and the solar radiation is 500 g. cal. per $cm.^2$ per day, first lay a straight edge between the given values on the X and t axes. Point A on the arbitrary S axis is an estimate of the combined effect of temperature and chlorophyll. Holding point A, the other end of the straight edge is moved to the 500 mark on the L axis. Point B, where this line intersects the W axis, is the estimate for production according to the combined effects of light, temperature and chlorophyll, in this case about 3.95 mgs. of oxygen per liter per week. For details in regard to the construction of alignment charts and their application to curvilinear correlations, the reader is referred to Bruce and Reineke (1931).

such. In a complete investigation, however, all relationships must be examined by some form of curvilinear technique. The effects of temperature are frequently non-linear, and this must be borne in mind in studying rates of growth, of feeding, of regeneration of nutrients, etc. The effect of inorganic nutrients and animal food supplies are probably also non-linear, and since the biological importance of a given substance increases when it becomes a limiting factor, the variations would be expected to have a greater effect, and perhaps a greater significance, when the substance in question is near the extinction point than when the total quantity is large.

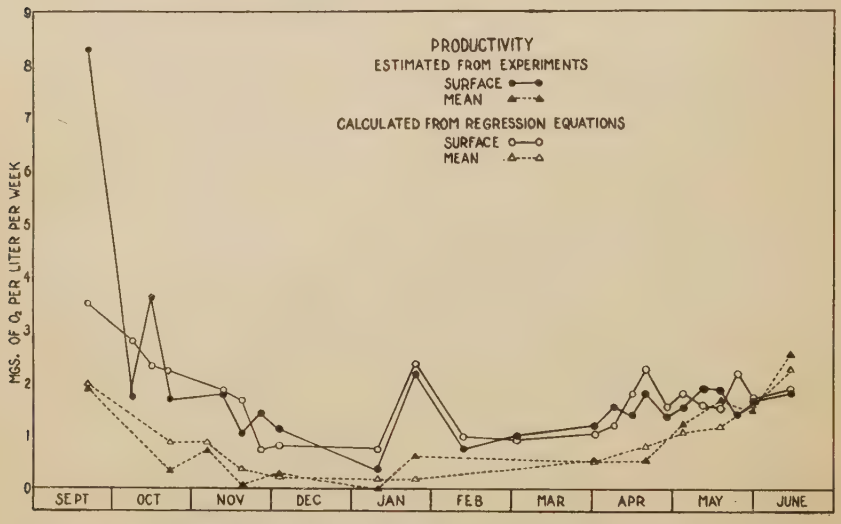

Figure 16. Productivity in Linsley Pond, 1937–38. Comparison of direct estimates from experiments with computations from regression equations.

The distinction just made between effect and significance requires some explanation:

The formula for a multiple regression equation is

$$W = \overline{W} + \beta wx \frac{\sigma w}{\sigma x}(X - \overline{X}) + \beta wy \frac{\sigma w}{\sigma y}(Y - \overline{Y})$$

where W is the dependent variable and X and Y are the independent variables, $\overline{W}$, $\overline{X}$, and $\overline{Y}$ are the means of the variables, βwx and βwy are the coefficients of partial correlation, and σx, etc., are the standard deviations. The total effect of X and Y on W is therefore dependent not only on the degree of relationship of the variations, but also on their total range. It is evident that without changing the biological significance, βwx, the effect

on W of a change of one unit in X may be altered by changing the standard deviation of either W or X. In other words, the slope of the regression curve is altered, but the degree of relationship is not. Therefore, in a curvilinear regression, which has a continual change of slope, there is a corresponding change in the ratio of standard deviations, which vary in different parts of the curve and are expressed in the equation only as an average. This can be corrected by including in the constant of each independent variable an expression for the form of the regression curve. It is evident that in such a case the constant is a better expression of all aspects of the relationship than is a simple statement of the biological significance.

On the other hand, it is possible for the biological significance of the relationship to vary in different parts of the curve. This can generally be determined by inspection of the plotted regression curve. The only method for dealing with the matter statistically is to divide the data into sections and treat them separately. This suggests what might be a useful index of the significance of a biologically important substance: namely, the curve for the relationship of the correlations to the quantity of substance at different levels of concentration.

THE MANIPULATION OF DATA

The qualitative and quantitative limitations of every set of data impose certain restrictions on statistical analysis. The ecologist is always impeded by analytical errors and by the technical difficulties of making simultaneous measurements of enough of the environmental factors. The quantity of data is never sufficient to extract all the needed information. It may be that a certain relationship can be fitted equally well by several different types of curves. When it is necessary to choose one of the curves for use in further analyses, it is a makeshift procedure, justifiable only for practical reasons. To use any one of the curves for biological theorizing is meaningless. A striking example of this is shown in a paper by Kavanagh and Richards (1934), who criticized the growth theories of Robertson and others by demonstrating that a given set of data might be fitted equally well by several equations of widely different mathematical character. They therefore concluded that it was unreasonable to use any one of these equations as the foundation for a theory of growth, merely because the equation fitted the data adequately.

It is evident, therefore, that a distinction should be made between a quasi-fit, which is good enough to determine correlation coefficients that are useful in making rough generalizations and in developing further analyses, and a real fit, where the form of the equation implies a theory of importance. The quasi-fit applies equally well to the cases just described, in which more than one type of curve could be fitted to a set of data, and to

the method of Bruce and Reineke, in which a curve is derived empirically from the data without regard to its mathematical nature.

It frequently happens that some minor factor is thought to have an ecological effect, but the statistical procedure is not delicate enough to reveal any mathematical significance. To attempt to carry an analysis beyond the point where the ingenuity of the investigator fails to produce significant results or to preserve a statistically insignificant factor in the final regression equation is useless.

As a general rule it is best to treat all the data together. Splitting it up into sections should not be attempted except for a specific purpose. An example of reasonable splitting might be postulated in the following way: In a previous section an estimate was made of phytoplankton respiration, which was probably accurate enough for the purpose, but which was subject to the assumptions that the respiration of the phytoplankton was the same as that of the seston as a whole, and that the variations of respiration with temperature are the same for all members of the association. With a much larger quantity of data, and with records of the quantity of bacteria, it would have been possible, and highly desirable, to separate the data into different temperature sections and determine the respiration of the different plankton groups in each temperature range by means of multiple correlations. But although splitting seems to be the only way of avoiding the unwarranted assumption, there is some danger that other factors which vary with temperature may influence the validity of the results. Corrections can be made for such inter-relations, but not only are the corrections less accurate because the smaller size of the groups (as compared with the total mass of data) increases the probable errors of the relationships, but also the labor of computation is much greater. For these reasons, the withdrawal of any small group from the main body of data is undesirable unless the required result cannot otherwise be obtained.

With all the above qualifications in mind, it is nevertheless true that correlations can serve a useful purpose even when the theoretical aspects of the relationship are not known. In handling graphical and tabular material, the biologist and those who read his work often have difficulty in evaluating relationships that lie on the borderline of significance. It may be stated almost axiomatically that a relation that is worth mentioning as having a possible validity is worth describing accurately, even though it is not known whether the causal significance is direct or indirect. The mathematical treatment removes much of the doubt about the relation, it saves time, and it lays a more firm basis for further study.

Hardy's work on the Hypothesis of Animal Exclusion (1935) is interesting in this respect, particularly since it is a theory that is likely to exert considerable influence on the whole of marine biology, regardless of whether it proves to be right or wrong. Much of the tabular material is of doubtful

mathematical significance. The relationships of the crude data on the relative quantities of phytoplankton and certain animal species (Table LII, p. 282) are statistically insignificant when treated on a simple linear basis. (It should be added, of course, that this statement implies no criticism of the hypothesis, since the table in question is not critical evidence, and since some of the later evidence appears to be mathematically sound.) It is obvious, however, that statistical techniques not only would provide a simpler method of examining the data, but would be essential in the final proof of the hypothesis, in order to evaluate the complicated indirect relationships that are involved.

THE ESTIMATION OF PHYTOPLANKTON PRODUCTIVITY

Finally, it appears desirable to discuss some of the theoretical aspects of plankton productivity, which, although they have no bearing on the main theme of the paper, are a logical development of the example used to illustrate the application of correlation techniques.

In stating productivity in terms of oxygen production, two useful purposes are served. The first one, which has been described, is the determination of the relative effect of different environmental factors. In the second place, it can be used to set the maximum and minimum limits of phytoplankton production.

In placing limits on production, it is necessary to assume that the values for oxygen production are valid (i. e., the estimate of respiration and the experimentally determined values for total oxygen production are accurate), that all the organic materials of which the oxygen figures are the indices are stored as food reserves or converted to protoplasm, and none is lost from the cell. Any errors in these assumptions would necessitate corrections in the estimates. With these qualifications in mind, conversion factors may be set up as follows:

1. If all the glucose produced is stored as such, the production is 0.94 g. for each gram of oxygen produced.

2. If the glucose is converted to lipid, and all the lipid is stored, the conversion factor is considerably lower, and variable according to the type of lipid produced. For purposes of convenience, the trigliceride of oleic, palmitic, and stearic acids is chosen (c. f. Bond, 1933), and it represents 0.34 g. of production per gram of oxygen.

3. The production of a gram of oxygen represents a fixation of 0.375 g. of carbon. According to the determinations of Birge and Juday (1922), the average carbon content of plankton is about 47.6% of the dry weight. Therefore, complete transformation of glucose carbon into plankton material would result in a production of 0.787 g. of plankton per gram of oxygen. This figure represents both stored food and protoplasmic material. If the

former is largely lipid, complete transformation to protoplasm would produce a higher conversion factor. On the other hand, since the energy required for plant syntheses is obtained by consuming part of the stored carbon, and since this loss, being entirely unrelated to the deduction that was previously made for plant respiration, must be taken into account here, the figure just given appears to be a maximum estimate for conversion to normal plankton material.

4. Two methods were used in an attempt to obtain a practical estimate of the relationship between oxygen production and the actual amount of plant material produced. The first was purely experimental, a determination of the difference in the volume of phytoplankton in the light and dark bottles at the end of an experiment. Between April 15 and 20, 1938, the weight of plankton, estimated from the volume, increased from 0.726 mg. per liter to 1.230 mgs. in the light bottle, and in the black bottle it decreased to 0.626 mg. The difference between these values was 0.604 mg. for the five day period, or 0.845 mg. per week. At the same time the oxygen production was 1.60 mgs. per liter per week and the estimated productivity in terms of oxygen was 1.35 mgs. Therefore, 0.625 g. of plankton was produced per gram of 0_2. This is 79% of the value obtained in estimate 3. Between October 1 and 9 the estimated weight of plankton produced was 1.347 mgs. per liter. The productivity in terms of oxygen production was about 3.70 mgs., although the exact amount is doubtful because two experiments overlapped at this time and the figure given is an average. According to this estimate, the weight of the phytoplankton produced is 0.364 g. per gram of 0_2, or 46% of estimate 3.

The other method of estimating productivity is based on the amount of utilization of nitrate in the Linsley surface water. The general principles of this technique have been applied by Atkins, Cooper, Harvey, and others to studies of marine plankton productivity, and while the result is always a minimum estimate, most of the error can be eliminated if the method is used cautiously. The changes that occur during short periods of time in the quantity of nitrate or other biologically important substance in the surface waters of a lake are largely due to a shift in the balance between the two opposing processes, consumption and regeneration. The plankton turnover can be estimated by the change in the quantity of the nutrient, but it does not approach the required degree of accuracy except when the ratio of consumption to regeneration is at a maximum. This occurs only at the beginning of a bloom. There were two occasions when the necessary conditions were fulfilled in Linsley. Between January 17 and 24, at the beginning of the mid-winter bloom, the decrease in nitrate was 0.071 mg. of *N* per liter, and from April 1 to 8, when the spring bloom started, it was 0.039 mg. Assuming that the plankton had a normal nitrogen content of 7%, this corresponds to a productivity of 1.01 mg. and 0.556 mg. of organic matter

respectively. The productivities in terms of oxygen were 2.16 mgs. and 1.20 mgs. per liter per week, so that the quantity of plankton produced per gram of oxygen was respectively 0.467 and 0.464 g. The correspondence in the two results is remarkably close, but it should not be given very much weight, for not only is the conversion factor of N to plankton a rough estimate which may vary from time to time, but also the April analyses of nitrate are not entirely above suspicion.

The mean of the four estimates is 0.48 g. of plankton per gram of oxygen, or 61% of estimate 3.

5. All the preceding estimates are higher than the values obtained by Gran and Ruud (1926) for marine plankton, which ranged from 0.1 to 0.2 g., with an average of 0.14 g. of plankton per gram of oxygen production. These values were obtained by comparing the oxygen production and the increase of plankton in growing cultures, and were therefore minimal because, as pointed out by Krogh, Lange, and Smith (1930), no allowance was made for the loss of organic matter by normal death and disintegration. On the other hand, this error might be rather small, and there remains the possibility that there might be a loss of glucose by secretion or that the loss due to organic syntheses might occasionally be very large. It therefore seems desirable to include the work of Gran and Ruud as an absolute minimum estimate.

The result of applying these estimates to the Linsley data is shown in Table III.

TABLE III

PHYTOPLANKTON PRODUCTION

(mgs. of organic matter produced per liter per week)

	Mean for entire period of observation		*Largest observed production*		*Smallest observed production*	
	Surface	Entire lake	Surface	Entire lake	Surface	Entire lake
Maximal estimate, gross conversion to glucose	1.73	0.87	7.80	2.56	0.36	0
Gross conversion to plankton of normal carbon content	1.45	0.73	6.53	2.14	0.30	0
Average estimate (Linsley experiments)	0.89	0.44	3.98	1.30	0.18	0
Gross conversion to lipid	0.63	0.32	2.82	0.92	0.13	0
Minimal estimate (Conversion factor of Gran and Ruud)	0.26	0.13	1.16	0.38	0.05	0

Taking the maximum and minimum estimates as setting the limits of production, we find that the mean phytoplankton production must lie between 0.56 and 3.81 crops per week at the surface, and between 0.32 and 2.17 in the entire lake. The difference between the maximum and minimum limits is rather large, and to use these particular conversion factors may be unduly cautious. Further work will probably show that the conversion to plankton of average carbon content and to lipid can be used as limits. According to the average estimate, based on the Linsley experiments, the production averaged 2.02 crops per week at the surface, with a total range of 0.4 to 6.3 crops. In the entire lake the mean was 1.10, and the variation was from 0 to 4.5 crops. But although the experimental estimate is useful as a rough approximation, it should not be regarded as particularly significant, for not only is it based on somewhat inadequate data, but also there is considerable evidence that the conversion factor varies according to environmental conditions. In a subsequent paper on the plankton of Long Island Sound, the author will discuss conversion factors more in detail and attempt to distinguish between production by mere accumulation of stored food reserves and production by the elaboration of protoplasmic materials.

ACKNOWLEDGMENTS

I should like to express my sincere gratitude to my colleagues who have read this paper and have made invaluable suggestions. I am particularly indebted to Mr. A. E. Parr, Mr. G. E. Hutchinson, Dr. Daniel Merriman, and Mr. M. D. Burkenroad.

SUMMARY

The application of correlations and regression equations to ecological work is discussed. It is shown that the statistical method achieves two practical results which cannot otherwise be obtained: First, it provides a quantitative measure of the significance of a relationship—the mathematical probability that the observed relationship has a causal connection; second, it permits the evaluation of complex inter-relationships.

It is pointed out that the coefficient of correlation is a purely mathematical symbol of relationship, a second order of factual material. The biological significance of a relationship coincides perfectly with the mathematical significance only in an ideal situation, in which all related factors are properly analyzed. In practical circumstances, therefore, the correlation should be regarded only as an approximation of the true relation, and any theory constructed upon it is qualified by the limitations of imperfect knowledge, just as any other scientific generalization is.

The problems involved in the application of correlations to ecology are illustrated by an analysis of the relationship of environmental factors to plankton production in Linsley Pond, a small lake in southern Connecticut. A note is added describing a method for estimating the maximum and minimum limits of plankton productivity.

REFERENCES CITED

1. Birge, E. A. and Juday, C.
1922. The inland lakes of Wisconsin. The plankton. Part I. Its quantity and chemical composition. Wis. Geol. Nat. Hist. Surv., Bull. *64:* 1–222.

2. Bond, R. M.
1933. A contribution to the study of the natural food-cycle in aquatic environments. Bull. Bingham Oceanogr. Coll. *4* (4): 1–89.

3. Bruce, D. and Reineke, L. H.
1931. Correlation alinement charts in forest research: a method of solving problems in curvilinear multiple correlation. U. S. Dept. Agr. Tech. Bull. *210:* 1–87.

4. Gran, H. H. and Ruud, B.
1926. Untersuchungen über die im Meerwasser gelösten organische Stoffe und ihr Verhältnis zur Planktonproduktion. Norske Vidensk Akad. Oslo I. M. N. Kl. No. 6.

5. Hardy, A. C. and Gunther, E. R.
1935. The plankton of the South Georgia whaling grounds and adjacent waters. 1926–1927. Part V. The Plankton community, the whale fisheries, and the hypothesis of animal exclusion (A. C. H.). Discovery Reports *11:* 273–360.

6. Kavanagh, A. J. and Richards, O. W.
1934. The autocatalytic growth curve. Amer. Nat. *68:* 54–59.

7. Krogh, A., Lange, E., and Smith, W.
1930. On the organic matter given off by algae. Biochem. J. *24:* 1666–1671.

8. Riley, G. A.
Limnological studies in Connecticut. Part III. The plankton of Linsley Pond. Ecol. Monogr. (*in press*).

SOME PHYSICAL FACTORS WHICH MAY INFLUENCE THE PRODUCTIVITY OF NEW ENGLAND'S COASTAL WATERS

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Of recent years physical oceanographers have gradually gained a more satisfactory understanding of the circulation problem. While it is true that most of this advance has resulted from studying the deep, oceanic currents, nevertheless, the newer principles also can be applied to the more difficult conditions found near the coast. More and more the physical investigators are working independently from the marine biologists. It is hoped that the following discussion of factors causing variations in the temperature and salinity of coastal waters off New England will demonstrate the biological significance of some of the modern physical investigations, and also perhaps help to bring the two branches of oceanography closer together again.

In general, coastal areas and oceanic areas differ fundamentally as far as the vertical distribution of salinity is concerned. Over the continental shelf the salinity usually increases with depth and, therefore, adds to the thermal stability of the water-column; while off shore in the deep water it decreases with depth and thus partly counteracts the stability resulting from the vertical temperature gradient. The fact that the salinity is at a maximum near the bottom on the continental shelf shows conclusively that the coastal circulation has an off shore component at the surface and an in shore component beneath.

The coastal waters, because of their relative freshness, are at most times of year less dense than the corresponding layer off shore and consequently a current is maintained which, for some reason not clearly understood, tends to have its greatest strength just outside the 100 fathom curve. This contour in general also corresponds roughly in position to the boundary between the two contrasting water-masses. The lateral transfer across the current, off shore at the surface and in shore along the bottom, supplies salt to the coastal waters which, because of land drainage, would otherwise gradually decrease in salinity. Off New England, coastal water can be defined as having a salinity of less than 34 $^{\circ}/_{\circ\circ}$, while higher values indicate an oceanic origin.

* Contribution No. 221.

If the continental shelf were more regular in depth and width, and in the absence of fluctuations in the oceanic currents, the necessary interchange between coastal and off shore waters would probably proceed at a steady, slow rate and a continuous current would flow along the 100 fathom curve. However, uneven bottom topography and major barriers such as Nantucket Shoals and Cape Hatteras tend to prevent a continuous and steady southward movement along the edge of our continental shelf. In addition, a number of deep troughs allow saline, off shore waters to work inward along the bottom in certain areas almost to the shore line. A notable example occurs in the Gulf of Maine where only a few miles east of Cape Cod salinities as high as $34.2°/_{oo}$ are found at the bottom. In general, however, wherever the continental shelf is straight and regular, as off New Jersey, the cross current components are less pronounced and there is less possibility of sudden fluctuations in the in shore conditions.

The exchange of waters across the 100 fathom curve is partly brought about by small scale mixing processes which result in a rapidly changing temperature-salinity correlation across the boundary between the coastal and the oceanic waters. In addition, there are frictional movements having a much larger scale which from time to time cause considerable volumes of relatively fresh water to move off shore near the surface. These intermittent movements are of course accompanied by a corresponding bottom invasion of more saline water, although the compensation can occur at some quite distant part of the continental shelf. From the standpoint of a biologist, the irregularity and strength of these large scale mixing mechanisms are of great significance, for they are capable of causing the death of large numbers of the population living near the edge of the shelf.

There are at least two different causes for the sudden, large scale, off shore movements of coastal water. During the autumn and early winter, especially if the inflow from rivers is for some reason particularly weak, the density of the waters over the continental shelf may become identical with that of the corresponding layer beyond the 100 fathom curve. In this case the coastal currents will die away and the winds are then free to force large masses of the relatively fresh, but cold, water off shore. It is indeed surprising that our coastal waters do not blow off to sea much more often than they do. Apparently their very great stability prevents the winds during most of the year from breaking down the local gradient currents. As long as a current is maintained along the 100 fathom curve, the in shore waters are in some way confined. No doubt bottom friction also plays an important role.

A second and more effective mechanism seems to depend on the large, frictionally driven eddies that from time to time develop along the northern edge of the Gulf Stream. These powerful eddies are often as much as 60 or 80 miles in diameter and thus can extend the whole distance from the edge

of the continental shelf to the Gulf Stream. They are known to occur north of the Gulf Stream at any point between Cape Hatteras and the Grand Banks, but they are most likely to be found off New York and off the Gulf of St. Lawrence. The structure of these eddies is as yet not well understood, nor is it known how long they persist. However, there is little doubt that, at least in their early stages, they can absorb large amounts of coastal

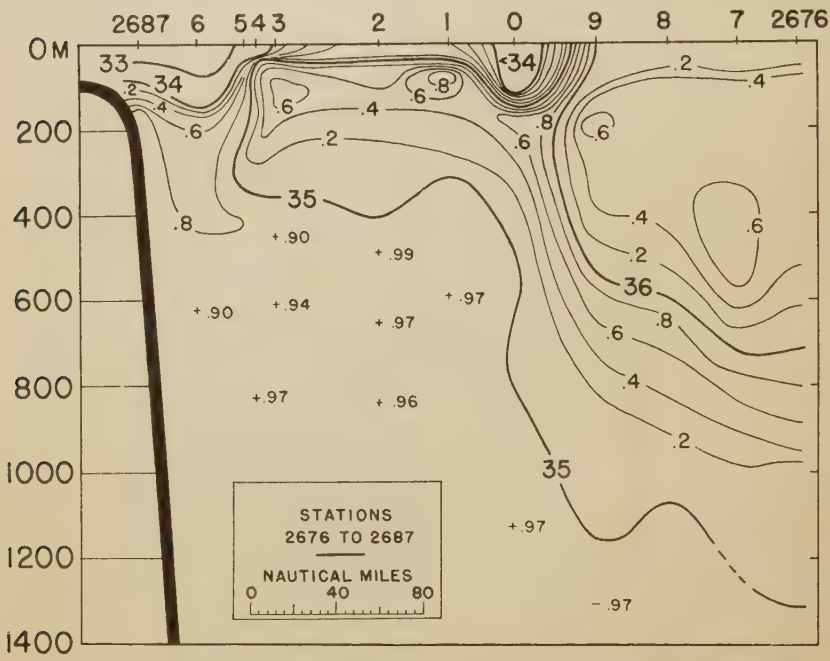

Figure 17. Salinity section, September 21–24, 1936, extending southeastward from the outer edge of Brown's Bank.

water. This they mix with Gulf Stream water and thus produce a surface layer of intermediate salinity, typical of this zone between the current and the edge of the continental shelf. This band, in which only weak permanent currents are found, is known as the slope water. The intermittent eddies can extend to considerable depths, but it is in the surface layer that the contrasting tongues of coastal and Gulf Stream water are observed. Thus tropical plankton becomes mixed with forms from our coastal shelf and both drift off slowly to the northeast, parallel to the Gulf Stream. No doubt the intermediate temperatures and salinities are unfavorable to most of these animals and in any case, the larvae of commercial fish, which happen to be

drawn into such eddies, will find themselves in deep water when the time comes to seek the bottom.

As yet no survey in the slope water area has had a sufficiently close station interval to reliably demonstrate the structure of these eddies. On several occasions, however, a single line of stations has crossed a body of almost undiluted coastal water as far off shore as the northern edge of the Gulf Stream (Figs. 17 and 18), but no direct observations are available to demonstrate whether such relatively fresh bodies of water are merely isolated pools or continuous tongues that have a connection with the waters

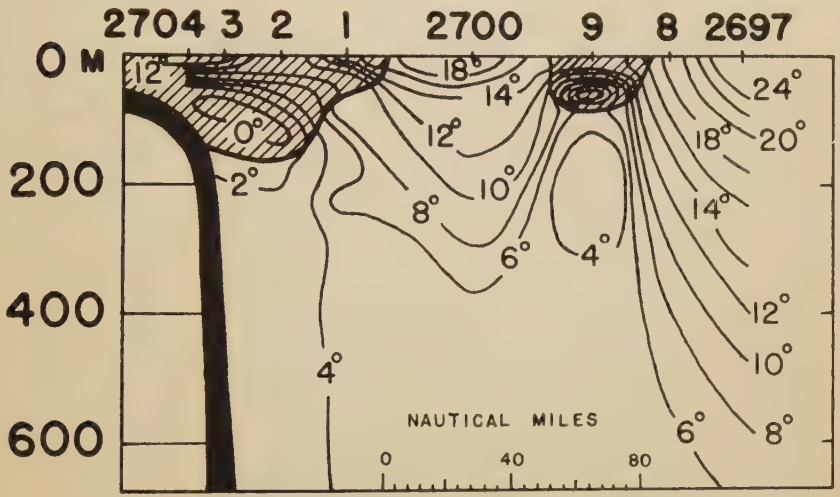

Figure 18. Temperature section, September 30–October 2, 1936, extending southwestward from the southern part of the Grand Banks. On this and subsequent sections the waters having salinities of less than 34°/oo are indicated by the cross-hatched areas.

over the continental shelf. In Fig. 17 almost pure coastal water can be seen at Station No. 2680 on the very edge of the Gulf Stream. This section was made in September 1936 on a line extending southeastward from Brown's Bank. Further to the eastward on the same cruise other bodies of coastal water were observed (Fig. 18) well beyond the continental slope. The positions of these stations, along with those of other sections made by the "Atlantis" within a two weeks period, are shown in Fig. 19. The latter diagram also demonstrates one possible interpretation of this survey. Admittedly the network of stations is so wide that any horizontal projection can be drawn in several different ways, but if any confidence is placed in the principles of isentropic analysis (Montgomery 1938), it seems likely that from Georges Banks eastward on this occasion there were several

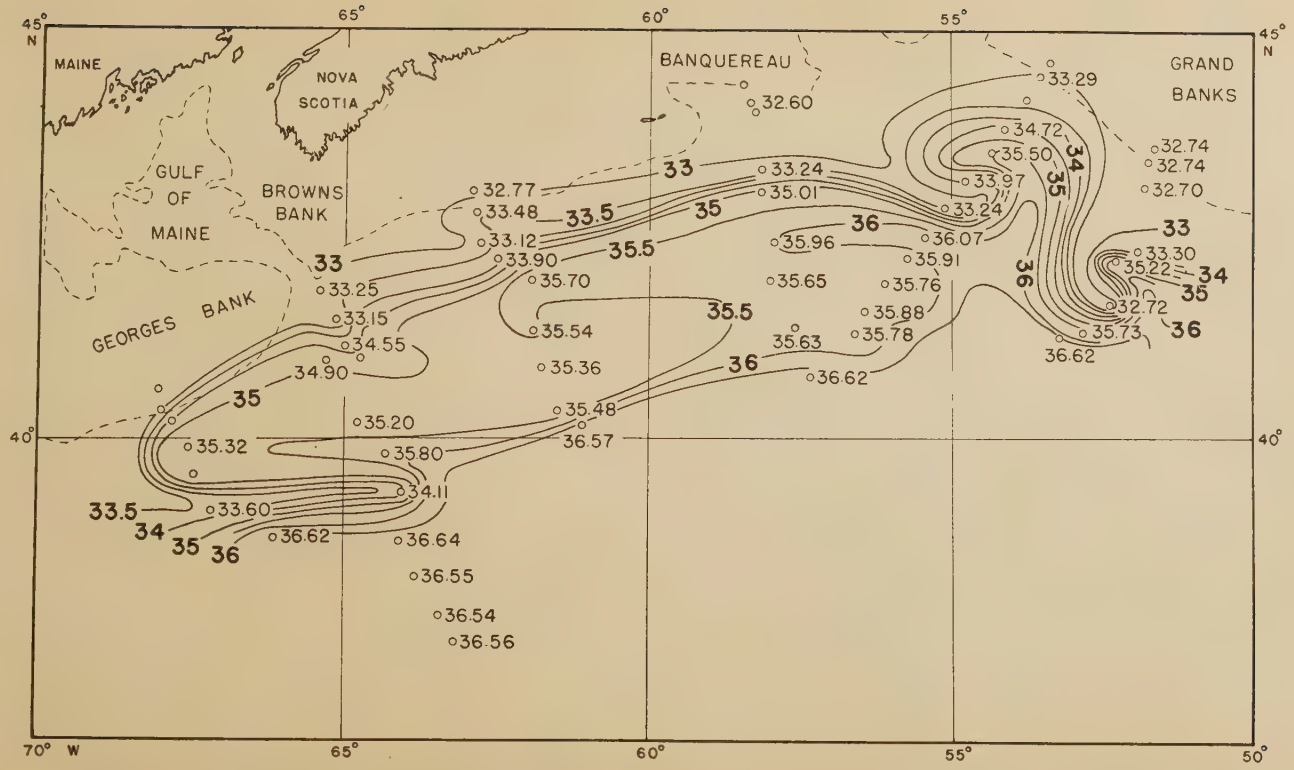

Figure 19. The distribution of salinity on the surface $\sigma_t = 26$ in the eastern slope water area, September 15–October 11, 1936.

tongues of coastal water being drawn off into the eddies in the manner indicated.

While a satisfactory understanding of these large scale mixing processes must await more specialized observations, there is already ample evidence that from time to time (Walford 1936) and in various ways a considerable amount of bank water is carried off shore in the surface layer of the slope water band. Whatever the mechanism, such movements will be destructive to the plankton thus removed from the fishing grounds. In addition, it can be argued on theoretical grounds that such eddies will be more numerous during periods when the Gulf Stream is decreasing in strength. Thus there is some hope that the Gulf Stream studies now being made (Iselin 1938) may provide a basis by which the fisheries investigator can judge, or even foretell, the relative proportion of a given year class that will be lost in this way.

The long period fluctuations in the strength of the Gulf Stream, if indeed they do exist, will have a more direct effect on bottom temperatures and salinities both at moderate depths on the continental slope and on the deeper parts of the shelf. In both cases the exact characteristics of the bottom water depend largely on the depth of the main thermocline layer in the slope water band, and this in turn is dependent on the volume of the Gulf Stream. When the current increases in volume, bottom temperatures and salinites will fall; when it weakens, they will rise.

In the deep water north of the current, when undisturbed by an eddy, the main thermocline layer (12°–6°) usually occupies the depths between 100 and 500 meters. South of the Gulf Stream the same isotherms and isohalines are found at the 500–1200 meter level (Fig. 17). In the extreme and most improbable case that the current should die away completely, they would continue at approximately these deeper levels right in to the continental slope. A decrease of 50% in the volume of the current would cause the slope water thermocline to deepen to intermediate depths. That only a slight fluctuation in the depth of the main thermocline layer in the off shore waters will have a marked effect on bottom conditions along the continental slope at depths between 100 and 400 meters can be seen by examining Figs. 20 and 21. These show in cross section the coastal water, and the superficial layers of the slope water and the Gulf Stream. The stations were occupied in early June, 1937 and 1938, on a line extending from Montauk Point to Bermuda. While on these two occasions bottom temperatures differed by only about 4° at depths between 150 and 200 meters, it is evident that a more marked change in the off shore currents could cause even wider fluctuations. In addition, these sections illustrate two different stages in the development of eddies. In one case (Fig. 21) the swirl was swift and contains mixed water at its center. In the other case

(Fig. 20), the slope water currents were moderate, but almost undiluted coastal water had been carried well off shore.

No doubt the long period changes in strength of the Gulf Stream are so gradual that the bottom living population is easily able to migrate up and down the continental slope and thus remain in water of suitable temperature and salinity. But it is also possible that from time to time sudden fluctuations in the off shore currents can cause rapidly changing bottom conditions that might even result in the death of the less mobile forms. It can be argued that the famous destruction of the tilefish in 1882 along the outer edge of the continental shelf between Delaware Bay and Nantucket Shoals

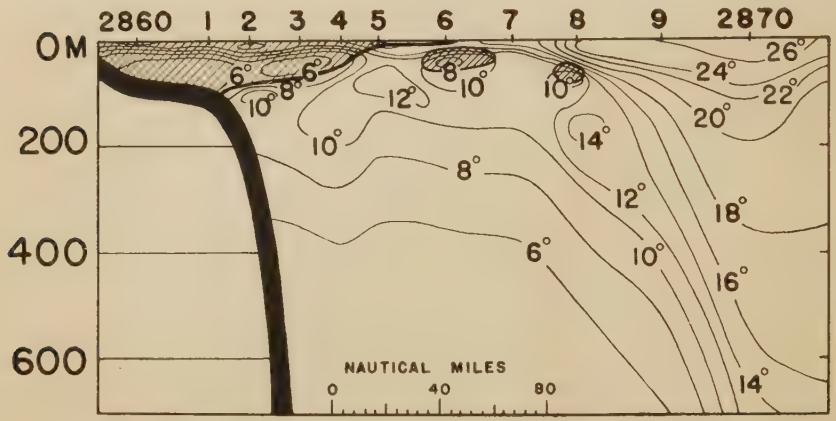

Figure 20. Temperature section, June 4–6, 1937, extending southeastward from Montauk Point, Long Island. Salinities less than 34°/oo are cross-hatched,

was due to this mechanism. In any case, there is a clear advantage for trawlers to obtain bottom temperatures when fishing in this critical zone.

Although much of the bottom water on top of the shelf, especially throughout the spring and summer months, is directly supplied from the upper part of the off shore, main thermocline layer, the possibility of sudden changes in its characteristics is more remote. However, the in shore bottom population, because of the relative flatness of the shelf, has much less chance of adjusting its depth to meet the rise and fall of the off shore isotherms and isohalines which must accompany the fluctuations of the Gulf Stream.

The principles which we have discussed thus far only have an important effect on life in the superficial layers near the edge of the continental shelf and on bottom forms on the outer half of the shelf or a short distance down the slope. Are there any influences which can cause significant variations

in the shallower waters nearer the coast? Obviously these must be much more under the control of the local climate and local mixing processes.

As mentioned already, throughout most of the year the waters over the continental shelf are extremely stable, both because of the off shore spread of relatively fresh water at the surface and because of the sharp, shallow thermocline layer which develops during the summer months. Under these conditions vertical turbulence is able to effect almost no transfer across the zone of maximum stability, just below the wind stirred, nearly homogeneous surface film. Thus as far as nutrients are concerned, the surface layer during the spring and summer cannot be supplied from be-

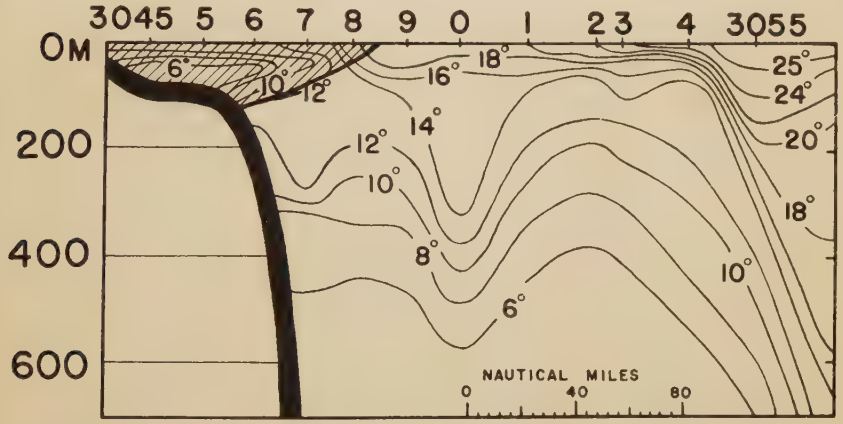

Figure 21. Temperature section, May 28–June 2, 1938, extending southeastward from Montauk Point, Long Island. Salinities less than 34°/oo are cross-hatched.

neath; and even at mid-depths the vertical transfer of essential chemicals must be extremely slow.

During the last few years physical oceanographers have begun to appreciate the important role which lateral turbulence plays in the sea (Rossby 1936, Parr 1938). Under the conditions of high stability often found in coastal waters, the isopicnal transport of nutrients should prove a most fruitful study. The vital biological role that lateral turbulence plays in the open ocean has been pointed out by Redfield (1936), and more recently (1939) he has studied this process from the point of view of certain planktonic forms in the Gulf of Maine. The chemicals which are in this way pumped up from mid-depths in the Sargasso Sea are transferred isopicnally across the Gulf Stream to the edge of the continental shelf. Thus as the bottom water penetrates landward, it carries with it a good supply of nutrients. But as long as the water-column is stable these cannot easily be

transferred to the photic zone, because very few surfaces of constant density cut the surface of the in shore waters.

It has been pointed out by Parr (1936) that in layers where the stability is high, the lateral turbulence is increased. This principle should not be overlooked by planktonologists. It suggests that over the continental shelf the lateral spread of plankton will be particularly marked in the transition layer between the relatively light surface water and the heavier bottom water.

The resistance which the extreme stability of our coastal water offers to vertical turbulence was nicely illustrated by the recent New England hurricane of September 1938. Fig. 22 shows the temperature-depth curves

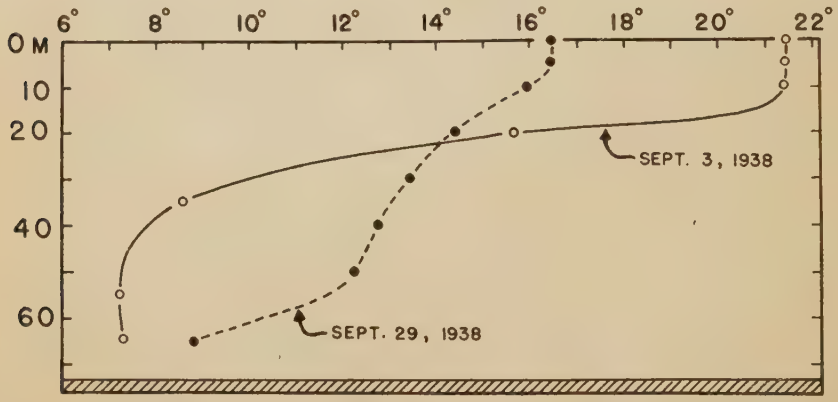

Figure 22. Temperature depth curves from a station about 20 miles southeast of Montauk Point, Long Island, before and after the New England hurricane of September 21, 1938.

from a station about 20 miles south of Montauk Point, Long Island, occupied on September 3 and again on September 29, 8 days after the gale. Although the center of the hurricane passed only a few miles west of this station, wind stirring was not able to break down the thermocline. However, turbulence did somewhat reduce the initial extreme temperature difference between surface and bottom.

The vital role which the breakdown of stability during the winter months plays in the fertility of the surface layer has often been emphasized, but it is less frequently mentioned that quite wide variations are possible from year to year in the effectiveness of the mid-winter stirring. Fig. 23 illustrates the cycle of temperature and salinity at a station in the western basin of the Gulf of Maine. At the surface the maximum cooling in this particular year (1934) came early in March. Although by mid-January the water was practically homogeneous down to 100 meters, a slight freshening

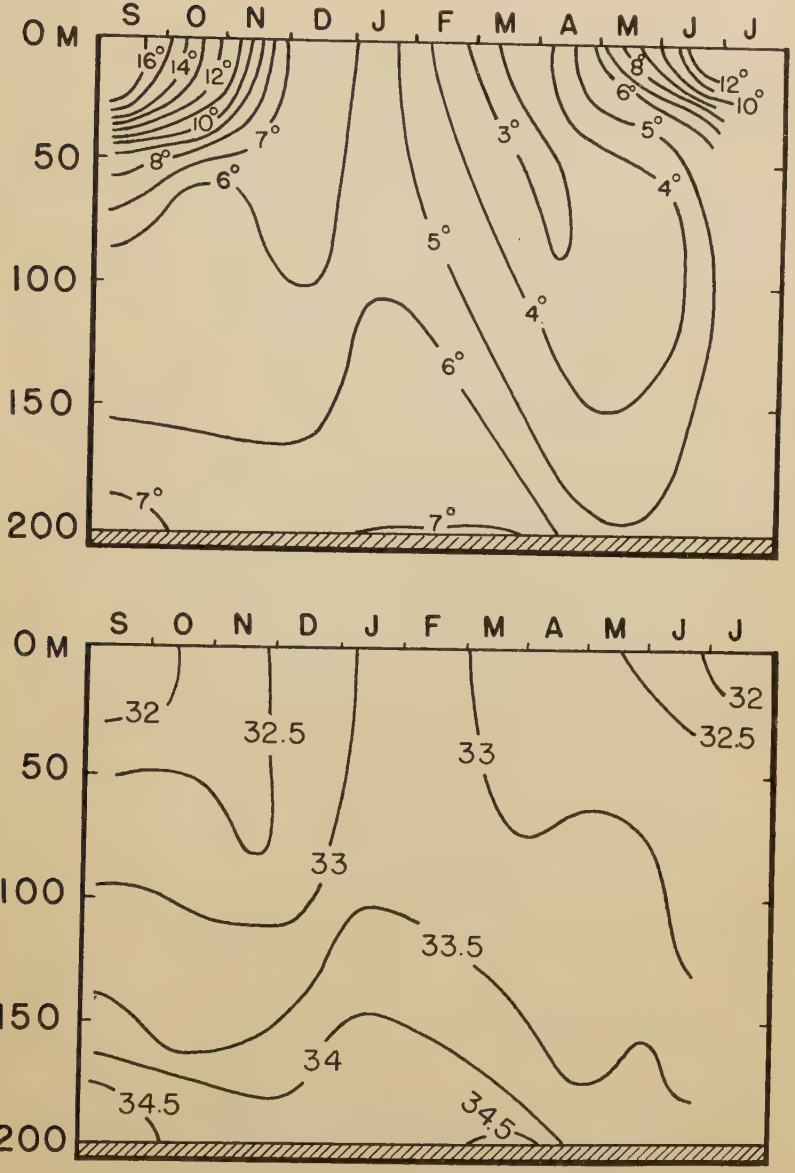

Figure 23. The changes in temperature and salinity, September 1933–July 1934, at a station in the western basin of the Gulf of Maine.

of the surface layer developed in February which retarded the downward transfer of minimum temperatures later in the winter. In addition, the temperature diagram shows that, because of the stability maintained by the vertical salinity gradient, turbulence did not bring minimum temperatures to depths below 100 meters until May. Meanwhile, the spring freshets and the early warming of the surface had begun to form a stable layer above. It is evident that the upward transfer of nutrients from the bottom would have been much more effective had the spring been delayed a few weeks.

That the winter overturn can be quite variable from year to year becomes clear when the factors which control vertical turbulence are considered. Obviously the colder the winter, the less stability will retard the deepening of the wind stirred surface layer. In addition, if the amount of river water spreading off shore at the surface is small, the stirring will also go deeper. Both factors tend to work together, for during severe winters much of the precipitation remains on the land in the form of snow and ice. In addition, during the short, critical period of minimum stability an especially heavy gale will do more to renew the fertility of the surface layer than all the winds of the other seasons. Finally, the longer the stability remains low, the better the chance of the mixing going to the bottom on the deeper parts of the continental shelf. These deeper gullies and basins contain a relatively rich supply of nutrients which will only be transported to the surface layer under conditions that are especially favorable to deep vertical mixing.

On the other hand, large areas of our continental shelf have a depth of less than 100 meters. In such regions the stability will completely break down early in the winter and there will be no chance of a supply of nutrients remaining on the bottom. In such cases the off shore spread of river water becomes the critical factor. If this occurs early and the winds are not strong enough to break through the fresher layer, then there is danger that before spring is well advanced the surface waters may become rather low in the essential chemicals. If such a winter is followed by an especially warm spring and summer, the plankton crop may be much below normal. The spectacularly poor mackerel season of 1937 was probably caused in this way. It has been pointed out (Settee 1937) that although on this occasion the stock of fish was sufficient, they were only caught during the summer months either very close to the shore, or on the outer banks. It seems likely (North American Council 1937) that only in these regions of strong tidal stirring could sufficient nutrients be maintained in the surface layer. Thus in order to secure food the mackerel were forced to change their usual migration route.

It is of course only natural that biological investigators should be interested in finding purely biological factors to explain the fluctuations in the fisheries. In such areas as Georges Banks, where, because of tidal stirring,

the water-column remains homogeneous throughout the year, the annual variations in temperature, salinity and fertility are indeed very small. In fact, this is probably one of the main reasons for the tremendous fertility of Georges Banks. The chemicals are continually being returned to the surface layer. However, as has already been explained, because this bank is near the edge of the continent, it is in a vulnerable position from the standpoint of the slope water eddies. It remains to be seen whether or not this purely physical mechanism is sufficiently variable from year to year to explain a significant part of the variations in the catch.

REFERENCES

1. Iselin, C. O'D.
   1938. Problems in the oceanography of the North Atlantic. Nature, Vol. 141, No. 3574.
2. Montgomery, R. B.
   1938. Circulation in upper layers of southern North Atlantic deduced with use of isentropic analysis. Papers in Physical Oceanography and Meteorology, Vol. VI, No. 2.
3. North American Council on Fishery Investigations.
   1937. Minutes of the Twenty-fourth Meeting, September 24–25, page 8.
4. Parr, A. E.
   1936. On the probable relationship between vertical stability and lateral mixing processes. Journal du Conseil, Vol. XI, No. 3.
5. Parr, A. E.
   1938. Isopycnic analysis of current flow by means of identifying properties. Journal of Marine Research, Vol. 1, No. 2.
6. Redfield, Alfred C.
   1936. An ecological aspect of the Gulf Stream. Nature, Vol. 138, p. 1013.
7. Redfield, Alfred C.
   1939. The history of a population of Limacina Retroversa during its drift across the Gulf of Maine. Biological Bulletin, Vol. LXXVI, No. 1.
8. Settee, O. E.
   1937. Mackerel failure is due to scarceness of food. Fishing Gazette, Vol. 54, No. 12.
9. Walford, Lional A.
   1938. Effects of currents on distribution and survival of the eggs and larvae of the haddock on Georges Banks. Bulletin of the U. S. Bureau of Fisheries, Vol. XLIX, No. 29.

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